Reg. No. : [| | | | | | | | | |]

T 3322

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

Fourth Semester

(Regulation 2004)

Computer Science and Engineering

MA 1252 — PROBABILITY AND QUEUEING THEORY

(Common to B.E. Part-Time Third Semester Regulation 2005)

Time : Three hours | Maximum : 100 marks

Use of statistical table is permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If the probability that a communication system will have high fidelity is 0.81 and the probability that it will have fidelity and high selectivity is 0.18, what is the probability that a system with high fidelity will also have high selectivity?

2. Given the probability density function $f(x) = \frac{k}{1+x^2}$, $-\alpha_1 < x < \alpha_1$, find k and C.D.F. $F(x)$.

3. If the probability is 0.10 that a certain kind of measuring device will show excessive drift, what is the probability that the fifth measuring device tested will be the first show excessive drift? Find its expected value also.

4. Let X be a uniform random variable over $[-1, 1]$. Find

   (a) $P(|X| < 1/3)$

   (b) $P(|X| \geq 3/4)$.

5. If X has mean 4 and variance 9, while Y has mean –2 and variance 5, and the two are independent, find

(a) $E(XY)$ (b) $E(XY^2)$.

6. Let X and Y be continuous RVs with J.p.d.f.

$$f(x, y) = \begin{cases} 2xy + \frac{3}{2}y^2, & 0 < x < 1,\ 0 < y < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

Find $P(X + Y < 1)$.

7. Define (a) Markov chain (b) Wide-Sense stationary process.

8. Let $\{X(t); t \geq 0\}$ be a Poisson process with rate λ. Find $E[X(t)X(t+\tau)]$, where $\tau > 0$.

9. In the usual notation of an M/M/1 queueing system, if $\lambda = 3$/hour and $\mu = 4$ /hour, find $P(X \geq 5)$ where X is the number of customers in the system.

10. Find $P(X \geq c + n)$ for an M/M/C queueing system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A box contains 5 red and 4 white balls. A ball from the box is taken out at random and kept outside. If once again a ball is drawn from the box, what is the probability that the drawn ball is red?

(ii) If the cumulative distribution function of a R.V. X is given by

$$F(x) = \begin{cases} 1 - \frac{4}{x^2}, & x > 2 \\ 0 & , x \leq 2, \end{cases}$$

find (1) $P(X < 3)$ (2) $P(4 < X < 5)$ (3) $P(X \geq 3)$.

(iii) A discrete R.V. X has moment generating function $M_X(t) = \left(\frac{1}{4} + \frac{3}{4}e^t\right)^5$.

Find $E(X), Var(X)$ and $P(X = 2)$.

Or

(b) (i) The p.d.f. of the samples of the amplitude of speech wave forms is found to decay exponentially at rate α, so the following p.d.f. is proposed.

$$f(x) = C e^{-\alpha|x|}, -\alpha_1 < x < \alpha_1 .$$

Find the constant 'C', and then find the probability $P(|X|<\nu)$ and $E(X)$.

(ii) Let X be a R.V. with $E(X)=1$ and $E(X(X-1))=4$. Find $Var\left(\frac{X}{2}\right)$ and $Var(2-3X)$.

(iii) If X is a continuous R.V. with p .d.f. $f(x)=\begin{cases} x & , 0 \le x < 1 \\ \frac{3}{2}(x-1)^2, & 1 \le x < 2 \\ 0 & , \text{otherwise,} \end{cases}$

find the cumulative distribution function $F(x)$ of X and use it to find $P\left(\frac{3}{2} < X < \frac{5}{2}\right)$.

12. (a) (i) If a R.V. X has geometric distribution, i.e., $P(X=x) = pq^{x-1}$, $x = 1, 2, 3$ where $q = 1-p$ and $0<p<1$, show that $P(X > x+y / X > y) = P(X > x)$.

(ii) Let X be a random variable with p.d.f. $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, $-\alpha_1 < x < \alpha_1$. Fnd the p.d.f. of the R.V. $Y = X^2$.

Or

(b) (i) Let the p.d.f. for X be given by $f(x)=\begin{cases} \frac{1}{2}e^{-x/2}, & x \ge 0 \\ 0 & , \text{otherwise.} \end{cases}$

Find (1) $P\left(X > \frac{1}{2}\right)$ (2) Moment generating function for X (3) $E(X)$ (4) $Var(X)$.

(ii) If X is any continuous R.V. having the p.d.f

$f(x)=\begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise,} \end{cases}$ and $Y = e^{X}$, find the p.d.f. of the R.V. Y.

13. (a) (i) Let X and Y have the joint p.d.f.

		X 0	1	2
Y	0	0.1	0.4	0.1
	1	0.2	0.2	0

Find (1) $P(X+Y>1)$ (2) the probability mass function $P(X=x)$ of the R.V. X (3) $P(Y=1/X=1)$ (4) $E(XY)$.

(ii) Suppose that orders at a restaurant are i.i.d. random variables with mean $\mu =$ Rs. 8 and standard deviation $\sigma =$ Rs. 2. Estimate (1) the probability that first 100 customers spend a total of more than Rs. 840, i.e. $P(X_1+X_2+\cdots+X_{100}>840)$ (2) $P(780<X_1+X_2+\cdots+X_{100}<820)$.

Or

(b) (i) Find $P(X>2/Y<4)$ when the joint p.d.f. of X and Y is given by

$$g(x, y) = \begin{cases} e^{-(x+y)}, & x \geq 0, y \geq 0 \\ 0 & , \text{otherwise.} \end{cases}$$

Are X and Y independent R.Vs? Explain.

(ii) If the joint p.d.f. of the R.Vs X and Y is given by $f(x, y) = \begin{cases} 2, & 0<x<y<1 \\ 0, & \text{otherwise,} \end{cases}$ find the p.d.f. of the R.V. $U = \dfrac{X}{Y}$.

14. (a) (i) Let $X(t)$ be a Poisson process with arrival rate λ. Find $E\left\{(X(t)-X(s)^2\right\}$ for $t>s$.

(ii) Let $\{X_n; n=1, 2, 3, \cdots\}$ be a Markov chain on the space $S=\{1, 2, 3\}$ with one step transition probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$.

(1) Sketch the transition diagram.

(2) Is the chain irreducible? Explain.

(3) Is the chain Ergodic? Explain.

Or

(b) (i) Consider a random process $X(t)$ defined by $X(t) = U\cos t + (V+1)\sin t$, where U and V are independent random variables for which $E(U) = E(V) = 0$; $E(U^2) = E(V^2) = 1$.

(1) Find the auto-covariance function of $X(t)$.

(2) Is $X(t)$ wide-sense stationary? Explain your answer.

(ii) Discuss the pure birth process and hence obtain its probabilities, mean and variance.

15. (a) (i) A concentrator receives messages from a group of terminals and transmits them over a single transmission line. Suppose that messages arrives according to a Poisson process at a rate of one message every 4 milliseconds and suppose that message transmission times are exponentially distributed with mean 3 ms. Find the mean number of messages in the system and the mean total delay in the system. What percentage increase in arrival rate results in a doubling of the above mean total delay?

(ii) Discuss the M/M/1 queueing system with finite capacity and obtain its steady-state probabilities and the mean number of customers in the system.

Or

(b) (i) A petrol pump station has 2 pumps. The service times follow the exponential distribution with mean of 4 minutes and cars arrive for service is a Poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What is the probability that the pumps remain idle?

(ii) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 min, determine (1) mean number of customers in the system L_S. (2) mean number of customers in the queue (3) mean waiting time in the system (4) mean waiting time in the queue.