Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐☐

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B.E/B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

Fifth Semester

(Regulation 2004)

Computer Science and Engineering

MA 1256 – DISCRETE MATHEMATICS

(Common to B.E. (Part – Time) Fourth Semester – Regulation 2005)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the premises P, Q and R are inconsistent prove that $\neg R$ is a conclusion from P and Q.

2. Prove by truth tables that $\neg(P \rightleftarrows Q) \Leftrightarrow (\neg P \vee \neg Q) \wedge (P \vee Q)$.

3. Let the Universe of discourse be $E = \{5,6,7\}$. Let $A = \{5,6\}$ and $B = \{6,7\}$. Let $P(x): x$ is in A; $Q(x): x$ is in B and $R(x,y): x + y < 12$ .Find the truth value of $((\exists x)(P(x) \rightarrow (Q(x)) \rightarrow R(5,6)$.

4. Give an example in which $(\exists x)(P(x) \rightarrow Q(x))$ is true but $((\exists x)P(x)) \rightarrow ((\exists x)(Q(x))$ is false.

5. If $R = \{(1,1),(1,2)(2,3)\}$ and $S = \{(2,1),(2,2),(3,2)\}$ are relations on the set $A = \{1,2,3\}$, verify whether $R \circ S = S \circ R$ by finding the relation matrices of $R \circ S$ and $S \circ R$.

6. In the following lattice find $(b_1 \oplus b_3) * b_2$:
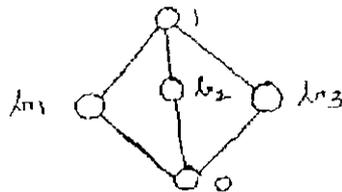
7. Find all the mappings from $A = \{1,2\}$ to $B = \{3,4\}$.

8. Let $h(x,y) = g(f_1(x,y), f_2(x,y))$ for all positive integers x and y, where $f_1(x,y) = x^2 + y^2, f_2(x,y) = x$ and $g(x,y) = xy^2$. Find $h(x,y)$ in terms of x and y.

9. Find a sub–group of order two of the group $(Z_8, +_8)$.

10. State Lagrange's theorem for finite groups.

PART B –– (5 × 16 = 80 marks)

11. (a) (i) Prove that $\left(\neg Q \to \neg P \right) \wedge \left(\neg R \to \neg Q \right) \Rightarrow (P \to R)$.   (6)

    (ii) Find the principal disjunctive and conjunctive normal forms of the formula $S \leftrightarrow \left(\left(\neg Q \vee \neg R \right) \to \neg P \right) \wedge (Q \vee R) \to P$.   (10)

Or

(b) (i) With out constructing the truth tables show that $A \vee C$ is not a valid consequence of the premises.

$$A \rightleftharpoons (B \to C), B \rightleftharpoons (\neg A \vee \neg C), C \rightleftharpoons (A \vee \neg B), B$$ (8)

    (ii) Using derivation process prove that

$$S \to \neg Q, S \vee R, \neg R, (\neg R \rightleftharpoons Q) \to \neg P.$$ (8)

12. (a) (i) Prove that $(\exists x)M(x)$ follows logically from the premises $(x)(H(x) \to M(x))$ and $(\exists x)H(x)$.   (8)

    (ii) Prove that $(\exists x)A(x) \to B \Leftrightarrow (x)(A(x) \to B)$.   (8)

Or

13.

14.

(b) (i) Prove that

$$(x)(P(x) \to Q(x)), (x)\big(R(x) \to \neg Q(x)\big) \Rightarrow (x)\big(R(x) \to \neg P(x)\big).$$ (8)

(ii) Prove that

$$(x)(H(x) \to A(x)) \Rightarrow (x)((\exists y)(H(y) \wedge N(x,y)) \to (\exists y)(A(y) \wedge N(x,y))).$$ (8)

13. (a) (i) If R is an equivalence relation on a set A, prove that $[x] = [y]$ if only if xRy where $[x]$ and $[y]$ denote equivalence classes containing x and y respectively. (6)

(ii) In a lattice show that

$$a \le b \Rightarrow a * b = a.$$ (4)

(iii) Prove that every chain is a distributive lattice. (6)

Or

(b) (i) If D_{45} denotes the set of all divisors of 45 under divisibility ordering, find which elements have complements and which do not have complements. (8)

(ii) In any Boolean algebra show that

$$a = 0 \Leftrightarrow ab' + a'b = b.$$ (8)

14. (a) (i) If f and g are bijections on a set A, prove that $f \circ g$ is also a bijection. (6)

(ii) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$ and $h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 4 & 3 & 1 \end{pmatrix}$ are permutations on the set $A = \{1,2,3,4,5\}$ find a permutation g on A such that $f \circ g = h \circ f$. (6)

(iii) The Ackerman function $A(x,y)$ is defined by

$$A(x,y) = y+1; A(x+1,0) = A(x,1); A(x+1,y+1) = A(x, A(x+1,y)).$$
Find $A(2,1)$. (4)

Or

(b) (i) Let $a < b$. If $f : [a,b] \to [0,1]$ is defined by $f(x) = \dfrac{x-a}{a-b}$, prove that f is a bijection and find it inverse (Here $[a,b]$ and $[0,1]$ are closed intervals). (8)

(ii) If $f : A \to B$ and $g : B \to C$ are mappings such that $g \circ f : A \to C$ is onto , prove that g is onto. Also give an example in which $g \circ f$ is onto while f is not onto. (8)

15. (a) (i) Let S be a non–empty set and $\mathbf{P}(S)$ denote the power set of S. Verify whether $(\mathbf{P(S)}, \cap)$ is a group. (6)

(ii) If H_1 and H_2 are sub–groups of a group $(G, *)$, prove that $H_1 \cap H_2$ is a sub–group of G. (4)

(iii) If G_1 and G_2 are groups and $f : G_1 \to G_2$ is a homomorphism, prove that the kernel of f viz, ker f is a normal sub–group of G_1. (6)

Or

(b) Let

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Find

(i) The Hamming code generated by H.

(ii) Find the minimum distance of the code.

(iii) If 00110 is the received word find the corresponding transmitted code word. (8 + 4 + 4)

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