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M.C.A. DEGREE EXAMINATIONS: OCTOBER / NOVEMBER 2008

Second Semester

P07CA201: Mathematical Foundations of Computer Science

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions

PART A (20 × 1 = 20 marks)

1. For what value of λ, the system of equations

$x + 2y + -3z = 0,\ \ 3x + \lambda y - z = 0,\ x - 2y + z = 0$ possess a non-trivial solution.

A. $\lambda = -2$ B. $\lambda = 2$ C. $\lambda = 3$ D. $\lambda = 3$

2. The eigen values of A^2 where

$$A \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

A. 1, 4 B. 1, 9 C. 1, 16 D. 1, 25

3. The condition satisfied by a, b, c so that the following system of equations $x + 2y - 3z = a,\ \ 3x - y + 2z = b,\ x - 5y + 8z = c$ to have a solution is

A. $3a-2b+4c = 0$. B. $2a-b+c = 0$. C. $2a-b+7c = 0$. D. $4a-2b-c = 0$.

4. If $\lambda^3 - \lambda^2 + 5\lambda - 1 = 0$ is the characteristic equation of a matrix B, then find B^{-1} in terms of B and I.

A. $B^{-1} = B^2 - B + 5I$. B. $B^{-1} = B^2 - 5B + I$.

C. $B^{-1} = 5B^2 - B + 5I$. D. $B^{-1} = 5B^2 - B - I$.

5. Which of the following is not true.

A. $A \oplus B = (A-B) \cup (B-A)$. B. $A \oplus B = (A \cup B) - (A \cap B)$.

C. $A-(B \cap C) = (A-B) \cup (A-C)$ D. $A-(B \cap C) = (A-B) \cap (A-C)$.

6. The function $f : R \rightarrow R$ defined by $f(x) = 5x^2 + 7$, $x \in R$ is

A. One-one and onto. B. One-one but not onto.

C. Onto but not one-one. D. Neither one-one and nor onto.

7. The relation R defined on the set of integers defined by aRa iff $a^2 > 0$ is

A. Reflexive, symmetric and transitive.

B. Symmetric and transitive but not reflexive.

C. Reflexive and transitive but not symmetric.

D. Reflexive and symmetric but not transitive.

8. How many reflexive relations are there on a set with "n" elements?

A. $2^{n(n+1)}$ B. $2^{(n(n+1)/2)}$ C. $2^{n(n-1)}$ D. $2^{(n(n-1)/2)}$

9. The compound statement $p \rightarrow [q \rightarrow (p \wedge q)]$ is a

A. Tautology. B. Contradiction. C. Contingency. D. Absurdity.

10. Which of the following is not true?

A. $p \rightarrow q \Leftrightarrow \sim q \rightarrow \sim p$. B. $p \rightarrow q \Leftrightarrow \sim p \vee q$. C. $p \rightarrow q \Leftrightarrow \sim p \rightarrow \sim q$. D. $p \rightarrow q \Leftrightarrow \sim(p \wedge \sim q)$.

11. The negation of the compound statement $(p \vee q) \rightarrow r$ is

A. $\sim p \wedge (\sim q \vee r)$. B. $(\sim p \wedge \sim q) \vee r$. C. $\sim p \wedge \sim q \wedge r$. D. $\sim r \vee (p \vee q)$.

12. The expression "Given a positive integer, there is a greater positive integer" where P(x): x is a positive integer, G(x, y) : x is greater than y in symbolic form is

A. $(y)\,(\exists x)\,(P(x) \rightarrow G(x, y))$. B. $(x)\,(\exists y)\,(P(x) \rightarrow G(x, y))$.

C. $(y)\,(\exists x)\,(P(x) \rightarrow G(y, x))$. D. $(x)\,(\exists y)\,(P(x) \rightarrow G(y, x))$.

13. The language given by the production rules P= {S→aB, A→aB, B→bA, B→b} is

A. $L= \{a^n b^n / n \geq 1\}$ B. $L= \{(ab)^n / n \geq 1\}$. C. $L= \{a^n b^m / n>m\}$. D. $L= \{a^n b^m / m>n\}$.

14. The grammar for he language $L= \{ a^i b^j c^k / i+j= k,\ i \geq 0, j \geq 0\}$ is

A. S→aAc/B, B→bbBc, B→cB/∈. B. S→aSc/A, A →aBb, B→bc/c, C→b/∈.

C. . S→aSc/A, A →bAc/∈ . D. S→aAc/B, A→bbBc, B→cB/∈.

15. Which of the following grammar is ambiguous?

A. S→aSb, S→∈. B. S→aSb, S→∈.

C. S→SS, S→a, S→b. D. S→aA, A →bbA, A→c.

16. Which of the following language is not context free?

A. $\{a^n b^{2n} / n \geq 1\}$ B. $\{ a^n b^n / n \geq 1\}$. C. $\{ (ab)^n / n \geq 1\}$. D. $\{ a^n b^n c^n / n \geq 1\}$.

17. Which of the following strings is accepted by the automata given by the transitions $\delta(q_0, a) = q_0$, $\delta(q_0, b) = q_1$, $\delta(q_1, a) = q_0$, $\delta(q_1, b) = q_1$ and q_1 is the accepting state is

A. $a^m b^n$ B. $a^n b^m a^k$ C. $ab^n a$ D. $a^n b^m$.

18. The finite automation that accepts $L= \{(ab)^n / n \geq 0\}$ is

A. $\delta(q_0, a) = q_0$, $\delta(q_0, b) = q_2$, $\delta(q_1, a) = q_2$, $\delta(q_1, b) = q_0$, $\delta(q_2, a) = q_2$, $\delta(q_2, b) = q_2$ where q_0 is the accepting state.

B. $\delta(q_0, a) = q_0$, $\delta(q_0, b) = q_2$, $\delta(q_1, a) = q_2$, $\delta(q_1, b) = q_0$, $\delta(q_2, a) = q_2$, $\delta(q_2, b) = q_2$ where q_1 is the accepting state.

C. $\delta(q_0, a) = q_0$, $\delta(q_0, b) = q_2$, $\delta(q_1, a) = q_2$, $\delta(q_1, b) = q_0$, $\delta(q_2, a) = q_2$, $\delta(q_2, b) = q_1$ where q_2 is the accepting state.

D. $\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_2$, $\delta(q_1, a) = q_2$, $\delta(q_1, b) = q_1$, $\delta(q_2, a) = q_1$, $\delta(q_2, b) = q_2$ where q_2 is the accepting state

19. The language accepted by the Finite state automata $\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_0$, $\delta(q_1, a) = q_2$, $\delta(q_1, b) = q_0$, $\delta(q_2, a) = q_2$ and , $\delta(q_2, b) = q_2$ where q_2 is the accepting state is

A. { set of all strings with equal number of a's and b's}.

B. { set of all strings in which every a is followed by a b.}.

C. { set of all strings with even number of b's}.

D. { set of all strings with even number of b's}.

20. The language accepted by the Non-deterministic finite state automata with transitions $\delta(q_0, a) = \{q_1, q_2\}$, $\delta(q_1, a) = q_1$, $\delta(q_1, b) = q_2$, $\delta(q_2, b) = q_2$, where $\{q_0, q_2\}$ as the accepting states is

A. $\{\in, a\} \cup \{(ab)^m / m \geq 1\}$. B. $\{\in, a\} \cup \{a^n b^n / n \geq 1\}$.

C. $\{\in, a\} \cup \{a^{2n} b^n / n \geq 1\}$. D. $\{\in, a\} \cup \{a^m b^n / m, n \geq 1\}$.

PART B (5 × 16 = 80 marks)

21. (a) (i) For what value of λ and μ the simultaneous equations $x + y + z = 6$: $x + 2y + 3z = 10$: $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. (8)

(ii) Find the eigen values and eigen vectors of adj A given that the matrix (8)

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

(OR)

21. (b) (i) Use Cayley-Hamilton theorem to find the value of the matrix given by ($A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^2 - 2A + I$) if the matrix (8)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

(ii) Find the rank of the matrix (8)

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

22 (a) (i) If A, B, C, D are sets prove algebraically that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$. Give an example to support this result(8)

(ii)Let A= R- {3} and B= R-{1} and a function f: A →B is defined as $f(x) = (x-2)/(x-3)$ for $x \in A$. Find (i) if f is 1-1. (ii) if f is onto (iii) find f^{-1} if it exists. In case f^{-1} does not exist, give reasons why it does not exist. (8)

(OR)

(b) (i) If R is the relation on the set of integers such that $(a, b) \in R$ if and only if $3a + 4b = 7n$ for some integer n. Check whether R is an equivalence relation. (8)

(ii)Find the number of primes greater than 1 and not exceeding 100 by using the principle of inclusion and exclusion. (8)

23 (a) (i) Obtain the pdnf and pcnf of
$(P \rightarrow (Q \wedge R)) \rightarrow (\sim P \wedge (\sim Q \wedge \sim R)$. (8)

(ii) Prove that $(x)P(x) \vee (x)Q(x) \Rightarrow (x)(P(x) \vee Q(x))$. (8)

(OR)

(b) (i) Prove that $R \rightarrow \sim Q$, $R \vee S$, $S \rightarrow \sim Q$, $P \rightarrow Q$ are inconsistent (8)

(ii) Prove that $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ without using truth table (8)

24. (a) (i) Construct a grammar to generate $L = \{ a^r / r$ is a perfect square$\}$. (8)

(ii) Show that $L = \{ a^{n!} / n \geq 0\}$ is not regular . (8)

(OR)

(b) (i) Prove that the language $L = \{a^n b^n / n \geq 1\}$ is unambiguous. (8)

(ii) If L_1 and L_2 are context free languages then prove that $L_1 \cup L_2$, $L_1 L_2$ and L_1^* are context free languages. (8)

25 (a) (i) Let $M = (\{q_0, q_1, q_2\}, \{a, b\}, q_0, \delta, \{q_2\})$ be a non-deterministic finite state automata, where δ is given by $\delta(q_0, a) = \{q_0, q_1\}$, $\delta(q_0, b) = \{q_2\}$, $\delta(q_1, a) = \{q_1\}$, $\delta(q_1, b) = \{q_0\}$, $\delta(q_2, a) = \{q_0\}$, $\delta(q_2, b) = \{q_1, q_2\}$. Construct an equivalent DFA. (8)

(ii) Construct a finite automation recognizing L(G), where G is the grammar $S \rightarrow aS/bA/b$ and $A \rightarrow aA/bS/a$. (8)

(OR)

25. (b) (i) Design a DFA that accepts all strings over {0, 1} except those containing the substring 001. (8)

(ii) Find a regular grammar accepting the recognized by the finite automata M represented by the transition table.

State	a	B
q_1	q_3	q_2
q_2	q_3	q_4
q_3	q_4	q_2
q_4	q_4	q_4

Where q_1 is the starting state and q_4 is the accepting state. .(8)

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