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M.E. DEGREE EXAMINATION, MAY/JUNE 2007.

First Semester

CAD/CAM/Computer Aided Design/Engineering Design

CD 1602 A — FINITE ELEMENT ANALYSIS

(For candidates admitted from the year 2006-07 onwards)

(Regulation 2005)

Time : Three hours

Maximum : 100 mark

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. How is minimum number of degrees of freedom per node determined in a element? Give an example.

2. Distinguish between essential and non-essential boundary conditions.

3. Express the constitutive matrix for a plane strain condition.

4. Give the compact representation of shape function for a four-node quadrilateral element.

5. How are in an isoparametric element the constant strain and rigid body conditions met?

6. What is static condensation? State any one of its applications.

7. Name different types of dynamic analysis. Give one application for each.

8. Distinguish between consistent mass matrix and lumped mass matrix.

9. Express the governing equation for heat conduction in a solid body in cylindrical coordinate system.

10. State two applications where fluid structure interaction is involved.

11. (a) (i) Explain with an example how the node numbering scheme influences the band width of the global stiffness matrix. (5)

(ii) Determine the axial deformation of a varying cross-section member shown in Fig. 1 under its own weight. Use one quadratic element. (11)
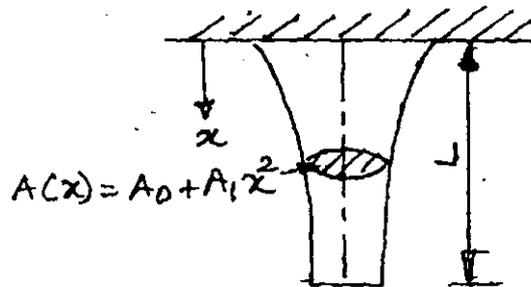

Fig. 1

Or

(b) A Steel rod of diameter D = 2 cm, length L = 5 cm, and thermal conductivity K = 50 W m^{-1} °C^{-1} is exposed to ambient air at T_a = 20°C with a heat transfer coefficient β = 100 W m^{-2} °C^{-1}. If the left end of the rod shown in Fig.2 is maintained at temperature T_0 = 320°C and the other end is insulated, determine the temperatures at distances 25 mm and 50 mm from the left end and the heat at the left end. The governing differential equation of the problem is

$$\frac{d^2\theta}{dx^2} = N^2\theta \text{ for } 0 < x < L$$

where θ = T - T_a, T is the temperature and N^2 is given by

$$N^2 = \frac{P\beta}{Ak} = \frac{\beta\pi D}{\frac{1}{4}\pi D^2 k} = \frac{4\beta}{kD}$$

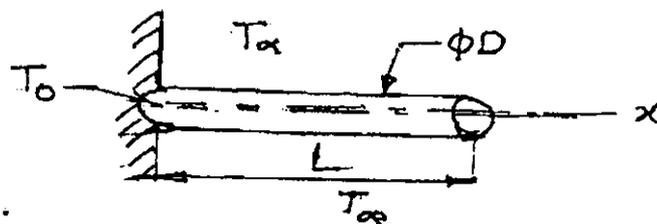The boundary conditions are $\theta(0)$ = T(0) - T_a = 300°C, $(d\theta/dx)$ = 0 at x = L. (16)

Fig. 2

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e numbering scheme
ss matrix. (5)

 cross-section member
 quadratic element.
(11)

5 cm, and thermal
nt air at T_a = 20°C
If the left end of the
'₀ = 320°C and the
t distances 25 mm
end. The governing

by

, $(d\theta/dx) = 0$ at
(16)

12. (a) If the nodal displacements of the rectangular element shown in Fig.3
$u_1 = 0.217$, $u_3 = 0.1870$, $u_2 = u_4 = 0.2232$. Compute u, $\partial u/\partial x$
$\partial u/\partial y$ at the point $(x, y) = (0.375, 0.375)$.

Fig.3
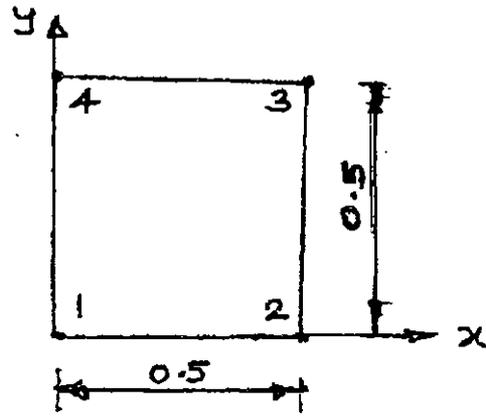
Or

(b) The coordinates of the nodes of a three-dimensional simplex element
given below :

Node number	Coordinates of the node		
	x	y	z
1	0	0	0
2	10	0	0
3	0	15	0
4	0	0	20

The components of displacements parallel to (x, y, z) at nodes 1,2,3, an
are found to be :

$u_1 = 0.0$; $u_2 = 0.01$; $u_3 = 0.02$; $u_4 = 0.0$,

$v_1 = 0.0$; $v_2 = 0.02$; $v_3 = 0.01$; $v_4 = 0.01$,

$w_1 = 0.0$; $w_2 = 0.01$; $w_3 = 0.01$; $w_4 = 0.01$

Determine the strains and stresses in the element. Take Youn
modulus as 210 GPa and Poisson's ratio as 0.3.

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13. (a) (i) Determine the 3 by 3 element stiffness matrix [k] of the bar element shown in Fig.4 having constant AE. Use an order 2 Gauss quadrature rule. (12)
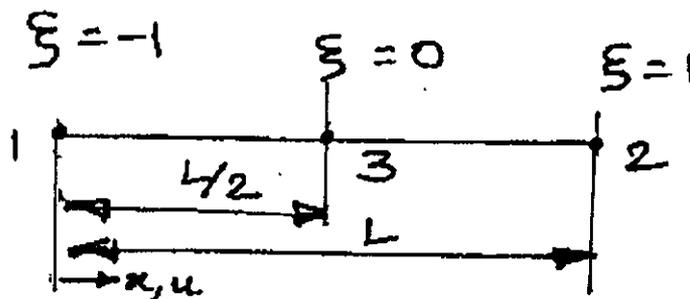

Fig. 4

(ii) Distinguish between subparametric and superparametric formulations. (4)

Or

(b) (i) A constant strain triangular element is shown in Fig. 5. The edge 1-2 is subjected to uniform pressure p_x along x-direction and edge 2-3 is subjected to uniform pressure along y-direction. Obtain the element nodal load vector. (10)
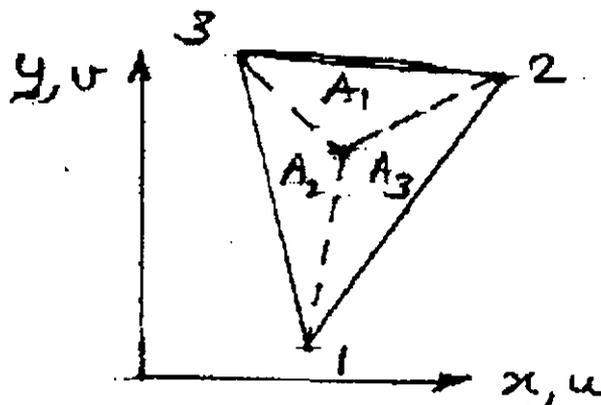

Fig. 5

(ii) Degrees of freedom θ_1 and θ_2 of the standard four-d.o.f. element shown in Fig.6 are to be eliminated. Determin condensed stiffness matrix [k].
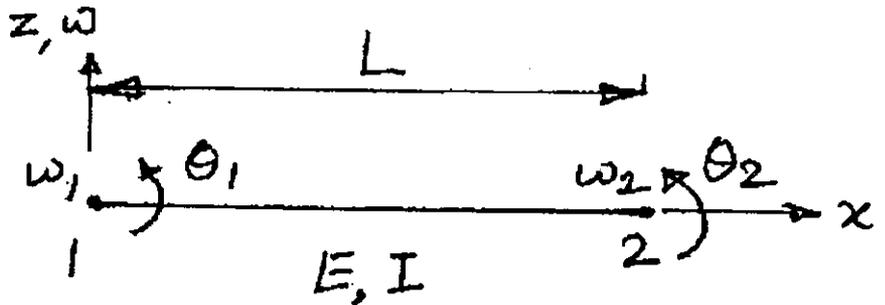

Fig. 6

14. (a) Using two-equal length finite elements, determine the natural cir frequencies of the solid circular shaft fixed at one end shown in Fig.7
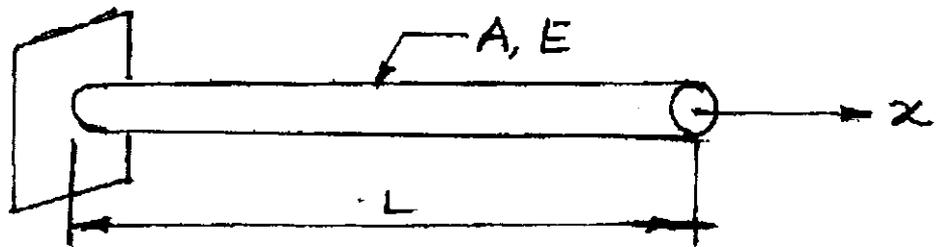

Fig. 7

Or

(b) The mass and stiffness matrices for a system are given below :

$$[m] = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} [k] = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 4 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

The initial displacement and velocity vectors are given by [0.3, 0.2, and [0, 5, 0] respectively. Determine natural frequencies and n shapes using Mode superposition method.

15. (a) A long bar of rectangular cross section, having thermal conductivity of $1.5 \ W\,m^{-1}\,{}^{\circ}C^{-1}$ is subjected to the boundary conditions shown in Fig. 8. Two opposite sides are maintained at a uniform temperature of 180°C; one side is insulated, and the remaining side is subjected to a convection process with $T_{\infty} = 25°C$ and $h = 50 \ Wm^{-2}\,{}^{\circ}C^{-1}$. Determine the temperature distribution in the bar. (16)
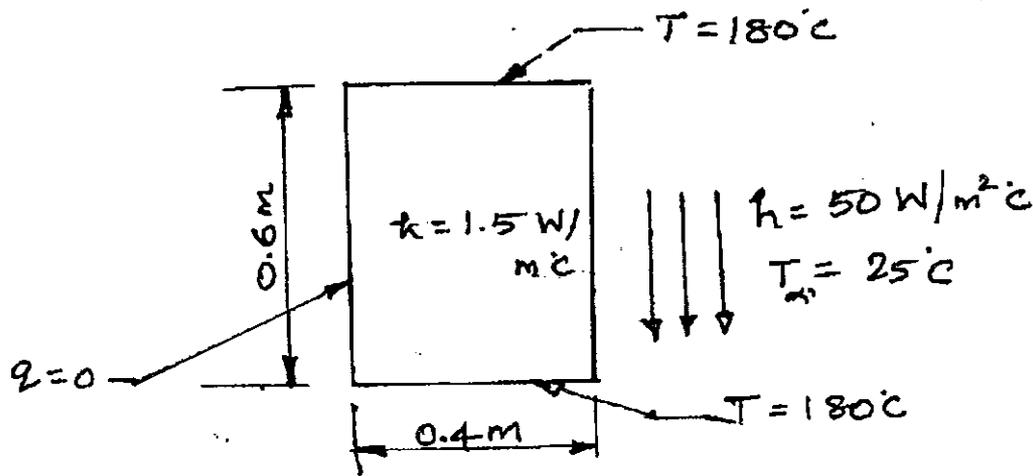

Fig. 8

Or

(b) ı Develop the necessary finite element equations for the analysis of two-dimensional steady flow (seepage) towards a well using linear rectangular isoparametric elements. (16)