B 2314

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Second Semester

Computer Science and Engineering

MA 035 — DISCRETE MATHEMATICS

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the equivalent of the conditional $p \to q$ using disjunction $(\vee)$.

2. If $S = \{-2, -1, 0, 1, 2\}$, determine the truth value of $\forall x \in S, |x|^2 \leq 3|x| - 2$.

3. Find the number of distinguishable permutations of the letters in SCIENCE.

4. Write an explicit formula for a_n, if $a_n = 3a_{n-1}$, and $a_1 = 2$.

5. In the group $\{1, 5, 7, 11\}$ under multiplication modulo, 12, what is the inverse of 5?

6. Define a ring.

7. Draw the Hasse diagram of $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$.

8. Find the value of the Boolean expression $x_1 x_2 \left(x_1 x_4 + x_2' + x_3 x_1' \right)$ if $x_1 = a, x_2 = 1, x_3 = b$ and $x_4 = 1$.

9. What is meant a pendent vertex in a graph?

10. Define a strongly connected graph.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the principal disjunctive normal form of $(P \wedge Q) \vee (\neg P \wedge R)$

(1) Using truth table

(2) Without using truth table. (4 + 4)

(ii) Let p, q, r be the following statements :

p : I will study discrete mathematics

q : I will watch T.V.

r : I am in a good mood.

Write the following statements in terms of p, q, r and logical connectives.

(1) If I do not study discrete mathematics and I watch T.V., then I am in a good mood

(2) If I am in a good mood, then I will study discrete mathematics or I will watch T.V.

(3) If I am not in a good mood, then I will not watch T.V. or I will study discrete mathematics.

(4) I will watch T.V. and I will not study discrete mathematics if and only if I am in a good mood. (4 × 2 = 8)

Or

(b) (i) What is meant by Tautology? Without using truth table, show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology. (2 + 6)

(ii) If there was rain, then travelling was difficult. If they had umbrella, then traveling was not difficult. They had umbrella. Therefore, there was no rain. Show that these statements constitute a valid argument. (8)

12. (a) (i) A computer password consists of a letter of English alphabet followed by 2 or 3 digits. Find the following

(1) The total number of passwords that can be formed

(2) The number of passwords in which no digit repeats. (4 + 4)

(ii) Using mathematical induction, show that for all positive integers n, $3^{2n+1} + 2^{n+2}$ is divisible by 7. (8)

Or

(b) (i) State the generalized pigeonhole principle. Using this, find the minimum number of students in a class to be sure that at least 3 of them are born in the same month. (4 + 4)

(ii) Solve the recurrence relation of the Fibonacci sequence of numbers $f_n = f_{n-1} + f_{n-2}, n > 2$ with the initial conditions $f_1 = 1, f_2 = 1$. (8)

13\. (a) (i) Show that the set of all nonzero real numbers in an Abelian group under * defined by $a * b = ab/2$. (8)

(ii) Let $(G *)$, and (H, Δ) between groups and let $g : G \to H$ be a group homomorphism from G to H. prove that

(1) $g(e_G) = e_H$ where e_G and e_H are the identities of $(G,*)$ and (H, Δ) respectively.

(2) $g(a^{-1}) = [g(a)]^{-1}$ for any $a \in G$. (4 + 4)

Or

(b) (i) Define left coset. Also show that the set of inverse of the a left coset is a right coset. (2 + 6)

(ii) Let f and g be the permutations of the elements of $\{1, 2, 3, 4, 5\}$ given by $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 1 & 2 \end{pmatrix}$. find gf^2g^{-1} and $g^{-1}fgf^{-1}$. (4 + 4)

14\. (a) (i) Let $(L, \leq)$ be a lattice. For any $a, b \in L$, prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ where * and $\oplus$ denote the operations of meet and join respectively. (8)

(ii) In any Boolean algebra, show that

(1) $a = b \Leftrightarrow ab' + a'b = 0$

(2) $a = 0 \Leftrightarrow ab' + a'b = b$. (4 + 4)

Or

(b) (i) Let $(L, *, \oplus)$ be a distributive lattice. For any $a, b, c, \in L$, prove that $(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c$. Deduce that in a distributive lattice if an element has a complement, it must be unique. (6 + 2)

(ii) Simplify the following Boolean expressions

(1) $(z' + x)(xy + z)(z' + y)$

(2) $(x + y)'(xy)'$. (4 + 4)

15. (a) (i) Define a complete graph K_n. Draw a complete graph K_6. What is the degree of each vertex in K_n? What is the total number of edges in K_n? (2 + 2 + 2 + 2)

(ii) Define an Euler path and show that if a graph G has more than two vertices of odd degrees, then there can be no Euler path in G. (2 + 6)

Or

(b) (i) Define Eulerian graph and Hamiltonian graph. Give an example of

(1) A graph which is Hamiltonian but not Eulerian

(2) A graph which is Eulerian but not Hamiltonian. (2 + 2 + 2 + 2)

(ii) Prove that a simple graph with n vertices and k components cannot have more than $(n-k)(n-k+1)/2$ edges. (8)
