B 2319

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Second Semester

Information Technology

MA 039 — PROBABILITY AND STATISTICS

Time : Three hours   Maximum : 100 marks

Statistical Tables are permitted

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the probabilities are 0.87, 0.36 and 0.29 that while under warranty, a new car will require repairs on the engine, drive train or both, what is the probability that a car will require one or the other or both kinds of repairs under the warranty?

2. If a random variable has the probability density $f(x) = \begin{cases} 2e^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$. Find the probability that it will taken on a value between 1 and 3. Find also the probability that it will take on a value greater than 0.5?

3. If the joint pdf of (X, Y) is $f(x, y) = 6e^{-2x-3y}$, $x \ge 0$, $y \ge 0$, find the marginal density of X and conditional density of Y given X?

4. If $X_1, X_2, \cdots X_n$ are Poisson variates with parameter $\lambda = 2$, use the central limit theorem to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + \cdots + X_n$ and $n = 75$?

5. A radio active source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4-min period.

6. If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a Poison arrives 2 min before the picture starts and if its takes exactly 1.5 min to reach the correct seat after purchasing the ticket, can he expect to be seated for the start of the picture?

7. A system has 5 identical components connected in series. If it is desired to have a system reliability of 0.95, how good should be the reliability of each component?

8. The density function of the time to failure of an appliance is $f(t) = \frac{32}{(t+4)^3}$, $t > 0$ is in years. Find the reliability function $R(t)$.

9. Distinguish between experimental and extraneous variables.

10. What do you mean by tolerance limits?

PART B — (5 × 16 = 80 marks)

11. (a) (i) In a bolt factory machines A, B, C manufacture respectively 25, 35 and 40 percent of the total. Of their output 5, 4 and 2 percent are defective bolts respectively. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B or C? (8)

(ii) The number of accidents in a year to taxi-drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with no accidents in a year. Also find the number of drivers with more than 3 accidents in a year. (8)

Or

12.

(b) (i) Find the mean and variance of the following density function :

$$f(x)=\begin{cases} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1<x<2 \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

(ii) A random variable X has the density function e^{-x} for $x \geq 0$. Prove that Chebychev inequality gives $P(|X-1|>2)<\frac{1}{4}$. Prove that the actual probability is e^{-3}. (8)

12. (a) (i) Find the marginal density functions of X and Y from the joint density function of X and Y.

$$f_{XY}(x,y)=\begin{cases} \frac{2}{5}(2x+3y) & 0 \leq x,\ y \leq 1 \\ 0 & \text{elsewhere} \end{cases} \qquad (6)$$

(ii) Two random variables X and Y have the joint density function

$$f_{XY}(x,y)\begin{cases} =x^2+\frac{xy}{3}, & 0 \leq x \leq 1;\quad 0 \leq y \leq 2 \\ =0 & \text{otherwise} \end{cases}$$

Prove that X and Y are not independent. Find the conditional density functions. Check whether the conditional density functions are valid. (10)

Or

(b) (i) If X and Y are independent random variables having density functions

$$f_1(x)=\begin{cases} 2e^{-2x} & x \geq 0 \text{ and} \\ 0 & x<0 \end{cases}$$

$$f_2(y)=\begin{cases} 3e^{-3y} & y \geq 0 \text{ and} \\ 0 & y<0 \end{cases}$$

Find the density function of their sum $U=X+Y$. (8)

(ii) The two regression lines are $4x - 5y + 33 = 0$ and $20x - 9y = 107$ and variance of $x = 25$. Find the means of x and y, the value of r and σ_y. (8)

13. (a) (i) Assume a random process $X(t)$ with four sample functions

$$x(t, s_1) = \cos t \qquad x(t, s_2) = -\cos t$$

$$x(t, s_3) = \sin t \qquad x(t, s_4) = -\sin t$$

which are equally likely? Prove that it is wide-sense stationary. (8)

(ii) A person owning a scooter has the option to switch over to scooter, bike or a car next time, with the probability of (0.3, 0.5, 0.2). If the transition probability matrix is $\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.3 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$. What are the probabilities of vehicles related to his fourth purchase? (8)

Or

(b) (i) The number of particles emitted by a radioactive source is Poisson distributed. The source emits particles at the rate of 6 per minute. Each emitted particle has a probability of 0.7 of being counted. Find the probability that 11 particles are counted in 4 minutes. (8)

(ii) A one-person barber shop has six chairs to accommodate people waiting for a haircut. Assume customers who arrive when all six chairs are full leave without entering the barber shop. Customers arrive at the average rate of 3/hr and spend an average of 15 min in the barber chair. What is the probability that a customer can get directly into the barber chair upon arrival? What percentage of time is the barber idle? (8)

14. (a) (i) Determine the MTTF for a mission time for 1000 hours life if the test data on 10 such components gave items to fail as shown in table.

Component number :	1	2	3	4	5	6	7	8	9	10
Time to failure in hours :	807	820	810	875	900	837	850	790	866	815

Also find the reliability. (8)

(ii) The life length of a device is exponentially distributed. It is found that the reliability of the device for 100 hour period of operation is 0.90. How many hours of operation is necessary to get a reliability 0.95? (8)

Or

(b) (i) Determine the average life length of a component obeying normal failure law with standard deviation of 10 hours if the operational reliability is to be 0.99 for an operation period of 100 hours. (8)

(ii) The life time in hours of a component is a random variable X which follows a Weibull distribution with $\alpha = 0.1$, $\beta = 0.5$. Obtain mean life time of these components and also calculate the probability that such a component will last more than 300 hours. (8)

15. (a) Three machines A, B, C gave the production of pieces in four days as below. Is there a significant difference between machines? (16)

A	17	16	14	13
B	15	12	19	18
C.	20	8	11	17

Or

(b) The table given below gives the measurements obtained in 10 samples. Construct control charts for mean and the range. Discuss the nature of control. (16)

Sample Number

	1	2	3	4	5	6	7	8	9	10
Measurements										
X	62	50	67	64	49	63	61	63	48	70
	68	58	70	62	98	75	71	72	79	52
	66	52	68	57	65	62	66	61	53	62
	68	58	56	62	64	58	69	53	61	50
	73	65	61	63	66	68	77	55	49	66
	68	66	66	74	64	55	53	57	56	75