B 2322

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Fourth Semester

Computer Science and Engineering

MA 040 — PROBABILITY AND QUEUEING THEORY

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If A and B are events in S such that $P(A) = 1/3$, $P(B) = 1/4$ and $P(A \cup B) = 1/2$. Find (a) $P(A \cap \overline{B})$ (b) $P(A/\overline{B})$.

2. Let X be a continuous R.V having the p.d.f. $f(x) = \begin{cases} \dfrac{2}{x^3}, & x \geq 1 \\ 0 & , \text{otherwise} \end{cases}$ Find the C.D.F for X.

3. The joint p.d.f. of a bivariate R.V. (X, Y) is given by $f(x, y) = \begin{cases} 4xy, & 0 < x < 1,\ 0 < y < 1 \\ 0 & , \text{otherwise} \end{cases}$ find $P(X + Y < 1)$.

4. State the central limit theorem.

5. Distinguish between wide sense stationary and strict sense stationary of the random process $X(t)$.

6. Find the autocorrelation function of a Poisson process $X(t)$ with rate λ.

7. Consider a Markov chain $\{X_n; n \geq 1\}$ with state space $S = \{0, 1, 2\}$ and one-step Transition probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Is the chain periodic? Explain.

8. In a reliability study of a system, it is found that the C.D.F for the random variable T is $F(t) = 1 - e^{-t/3} - e^{-t/6} + e^{-t/2}$. What is the reliability for the system? What is the MTTF for the system?

9. In a given M/M/1 queue, the arrival rate $\lambda = 7$ customers/hour and service rate h = 10 customers/hour. Find $P(X \geq 5)$, where X is the number of customers in the system.

10. What is the effect arrival rate for M/M/1/N queueing system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Box–I contains 1 white, 2 red, 3 green balls, box–II contains 2 white, 3 red, 1 green balls, Box–III contains 3 white, 1 red, 2 green balls. Two balls are drawn from a box chosen at random. These are found to be one white and one red. Determine the probability that the balls so drawn came from Box – II.

(ii) If X is a binomially distributed R.V. with $E(X) = 2$ and $\text{Var}(X) = \frac{4}{3}$, find $P(X = 5)$.

(iii) Apply Chebyshev's inequality to calculate $P(|X - 10| \geq 3)$ for a random variable X with $E(X) = 10$ and $\text{Var}(X) = 4$.

Or

(b) (i) A bag contains eight white and six red marbles. Find the probability of drawing two marbles of the same colour.

(ii) If X is uniformly distributed with $E(X) = 1$ and $\text{Var}(X) = 4/3$, find $P(X < 0)$.

(iii) Let X be an exponential R.V. with $E(X) = 1$. Find the probability density function of the R.V. $Y = -\log X$.

12. (a) (i) Determine the correlation coefficient between R.Vs X and Y whose j.p.d.f. is $f(x, y) = \begin{cases} x + y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$

(ii) If X and Y are two R.V.s having j.p.d.f

$$f(x,y) = \begin{cases} \frac{1}{8}(6 - x - y), & 0 < x < 2, 2 < y < 4 \\ 0 & \text{, otherwise} \end{cases}$$

Find (1) $P(X < 1, Y < 3)$ (2) $P(X + Y < 3)$ (3) $P(X < 1 | Y < 3)$.

Or

(b) (i) Find the conditional p.d.f. $f(y/x)$ and $E(Y/X=x)$ when the R.Vs X and Y have joint p.d.f $f(x, y) = \begin{cases} \lambda^2 e^{-\lambda x}, 0 \le x \le y \\ 0, \text{otherwise} \end{cases}$

(ii) Let X and Y be independent exponential R.Vs. with parameter 1. Find the j.p.d.f of $U = X+Y$ and $V = \dfrac{X}{X+Y}$ and deduce that ∇ is uniformly distributed on [0, 1].

13. (a) (i) Let $X_n = A\cos(n\lambda) + B\sin(n\lambda)$ where A and B are uncorrelated random variables with $E(A) = E(B) = 0$ and $\text{Var}(A) = \text{Var}(B) = 1$. Show that X_n is covariance stationary.

(ii) Discuss the pure birth process and hence obtain its probabilities $P_{n(t)}, n = 1, 2, 3\ldots,$ mean and variance.

Or

(b) (i) The number of cups of coffee ordered per hour at cafteria follows a Poisson process with an average of 30 coffee per hour being ordered (1) find the probability that exactly 60 coffees are ordered between 10 P.M. and 12 midnight (2) find the mean and standard deviation of the number of coffee ordered between 9 P.M. and 1 A.M. (3) Find the probability that the time between two consecutive order between 1 and 3 minutes.

(ii) Let $\{N(t); t \ge 0\}$ be a renewal process with C.D.F $F(t)$. Show that $P\{N(t) = n\} = F^{(n)}(t) - F^{(n+1)}(t)$ and $E(N(t)) = \sum_{n=1}^{\alpha_1} F_{(t)}^{(n)}$ where $F^{(n)}(t)$ is the n–sold convolution of $F(t)$ with itself.

14. (a) (i) Consider a Markov chain $\{X_n; n \ge 1\}$ with state space $S = \{1, 2\}$ and one–step transition probability matrix. $P = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$.

(1) Is chain irreducible?

(2) Find the mean recurrence time of states '1' and '2'

(3) Find the invariant probabilities.

(ii) Discuss the preventive maintenance of the system and hence obtain MTTSF for a system having n identical units in series with exponential failure–time distribution.

Or

(b) (i) Discuss the reliability analysis for 2–unit parallel system with repair.

(ii) The life length of a device is exponentially distributed. It is found that the reliability of the device for 100 hour period of operation is 0.90. How many hours of operation is necessary to get a reliability of 0.95?

15. (a) (i) An average of 10 cars per hour arrive at a single–server drive–in teller. Assume that the average service time for each customer is 4 minutes, and both inter arrival times and services are exponential (1) what is the probability that the teller is idle (2) what is the average number of cars waiting in the line for the teller? (3) what is the average amount of time a drive–in customer spends in the bank parking (4) on the average, how many customer per hour will be served by the teller?

(ii) Obtain the steady state probabilities for M/M/1/N FCFS queueing model.

Or

(b) (i) A one–man barber has a total of 10 seats. Inter arrival times are exponentially distributed, and an average of 20 prospective customers arrive each hour at the shop. Those customers who find the shop full do not enter. The barber takes an average of 12 minutes to cut each customer's hair. Haircut times are exponentially distributed (1) on the average how many haircut per hour will the barber complete? (2) on the average, how much time will be spent in the shop by a customer who enters?

(ii) Automatic car W has facility operates with only one bay. Cars arrive according to a Poisson distribution, with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes, determine L_s, L_q, W_s and W_q.