O.C.

E 5078

M.E. DEGREE EXAMINATION, MAY/JUNE 2007.

First Semester

Structural Engineering

MA 145 — APPLIED MATHEMATICS

(Regulation 2002)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If u is a function of x and t, find the Fourier cosine transform of $\frac{\partial^2 u}{\partial x^2}$ in $0 \le x \le l$.

2. Find the Laplace transform of the equation $\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

3. Write down the properties of harmonic functions.

4. Write down the equation which gives the temperature distribution of the plate in the steady state.

5. What is Euler-Lagrange's differential equation in variational problem?

6. Find the extremel of the functional $V[y(x)] = \int_{x_0}^{x_1} (16y^2 - y''^2 + x^2)\,dx$.

7. If $f(x) = \frac{1}{\pi(1+x^2)}$, $-\infty < x < \infty$, find the distribution function $F(x)$.

8. If each of the two variables $X \& Y$ takes two values 0, 1 with positive probabilities, then $r(X,Y) = 0 \Rightarrow X \& Y$ are independent.

9. Write down the equation of the plane of regression of X_1 on X_2 and X_3.

10. Prove that for a normal distribution, sample mean is more efficient estimates for μ than sample median.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the extremel of the functional $\int_0^{\pi}(4y\cos x + y'^2 - y^2)dx$ given that $y(0) = 0$; $y(\pi) = 0$. (8)

(ii) Determine the extremel of the functional $v[y(x)] = \int_0^{\pi/2}(y''^2 - y^2 + x^2)dx$ that satisfies the conditions $y(0) = 1$, $y'(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$, $y'\left(\frac{\pi}{2}\right) = -1$. (8)

Or

(b) Find an approximate solution to the problem of the minimum, of the functional

$$v[y(x)] = \int_0^1 (y'^2 - y^2 + 2xy)dx$$

$y(0) = 0 = y(1)$ by Ritz method and compare it with the exact solution. (16)

12. (a) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ subject to the initial conditions $y(x,0) = f(x)$, $-\infty < x < \infty$, $\frac{\partial y}{\partial t}(x,0) = g(x)$ and the boundary conditions $y(x,t) \to 0$ as $x \to \pm\infty$. (16)

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, satisfying the boundary conditions $u(0,t) = 0$ and $u(x,t) \to 0$ as $x \to \infty$ and the initial condition $u(x,0) = f(x)$, $x > 0$. (16)

13. (a) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $y \geq 0$ subject to the boundary conditions $u(x,0) = f(x)$, $-\infty < x < \infty$ and $u(x,y) \to 0$ as $y \to \infty$. (16)

Or

(b) Solve the two dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $x \geq 0$, $y \geq 0$ and $u(0,y) = 0$; u is bounded; $\frac{\partial u}{\partial y}(x,0) = f(x)$. (16)

14. (a) (i) X and Y are two random variables having the joint density function $f(x,y) = \frac{1}{27}(2x + y)$; wave x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y for $X = x$. (8)

(ii) Obtain the lines of regression and find the coefficient of correlation from the following : (8)

x :	1	2	3	4	5	6	7
y :	9	8	10	12	11	13	14

Or

(b) (i) X is a random variable whose density function is

$f(x) = Ae^{-x}$, $0 < x < \infty$
$= 0$ otherwise

Find the value of (1) A, (2) Mean of X, (3) Variance of X, (4) third moment about the mean and kurtosis. (8)

(ii) Find the moment generating function of a normal distribution and hence find M.G.F. of a standard normal variate. (8)

15. (a) Show that $1 - R^2_{1.23} = (1 - r^2_{12})(1 - r^2_{13.2})$

Deduce that

(i) $R_{1.23} \geq r_{12}$

(ii) $R^2_{1.23} = r^2_{12} + r^2_{13}$ if $r_{23} = 0$

(iii) $1 - R^2_{1.23} = \dfrac{(1-\rho)(1+2\rho)}{1+\rho}$, provided all the coefficients of zero order are equal to ρ.

(iv) If $R_{1.23} = 0$, X_1 is uncorrelated with any of other variables (i.e.) $r_{12} = r_{13} = 0$. (16)

Or

(b) (i) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance. (8)

(ii) Estimate α and β in the case of Pearson's type distribution by the method of moments where $f(x;\alpha,\beta) = \dfrac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, $0 \le x < \infty$. (8)

Time :

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.