G 6313

M.E. DEGREE EXAMINATION, MAY/JUNE 2007.

First Semester

Structural Engineering

MA 1602 — APPLIED MATHEMATICS

(Common to M.E. – Soil Mechanics and Foundation Engineering)

(Regulation 2005)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace transform of $f(t) = \sin^3 2t$.

2. Find the inverse Laplace transform of $\dfrac{1}{(s+1)^2}$.

3. Write the necessary condition for $\int_{x_1}^{x_2} f(x, y, y')\,dx$ to be an extremum.

4. What is the necessary condition for $\int_{x_1}^{x_2} f(x, y, y', y'')\,dx$ to be extremum.

5. Define random variable. Classify it.

6. Define correlation coefficient of random variables X and Y.

7. State any two properties of Maximum likelihood estimators.

8. State Invariance property of maximum likelihood estimators.

9. Define Fuzzy variable.

10. Define Fuzzy likelihood function.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find $y(t)$ which satisfies the equation $\frac{dy}{dt}+4y+5\int_0^t y\,dt=e^{-t}$ when $y(0)=0$. (8)

(ii) Find the inverse Laplace transform of the function $\log\left(1+\frac{w^2}{s^2}\right)$. (8)

Or

(b) (i) Solve $\frac{dx}{dt}+y=\sin t;\ \frac{dy}{dt}+x=\cos t$ with $x=2$ and $y=0$ when $t=0$. (8)

(ii) Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\left(\frac{\partial^2 u}{\partial x^2}\right), x>0, t>0$ where $u(x,0)=0, u_t(x,0)=0, x>0$ and $u(0,t)=F(t)$, $\lim_{x\to\infty}u(x,t)=0, t>0$. (8)

12. (a) (i) Find the extremals of the functional $\int_{x_0}^{x_1}\left(y'^2/x^3\right)dx$. (6)

(ii) Solve the boundary value problem $y''-y+x=0\ (0\le x\le 1)$, $y(0)=y(1)=0$ by Rayleigh–Ritz method. (10)

Or

(b) (i) On which curve the functional $\int_0^{\pi/2}(y'^2-y^2+2xy)dy$ with $y(0)=0$ and $y(\pi/2)=0$, be extremized? (8)

(ii) Find the curves on which the functional $\int_0^1[(y')^2+12xy]\,dx$ with $y(0)=0$ and $y(1)=1$ can be extremised. (8)

13. (a) (i) A random variable X has the following probability distribution

X:	−2	−1	0	1	2	3
$p(X)$:	0.1	K	0.2	$2K$	0.3	$3K$

(1) Find K

(2) Evaluate $P(X<2)$ and $P(-2<X<2)$

(3) Find the cumulative distribution function of X and

(4) Evaluate the mean of X. (8)

14.

(ii) Find the regression equation of X_1 on X_2 and X_3 given the following results :

Trait	Mean	Standard deviation	r_{12}	r_{23}	r_{31}
X_1	28.02	4.42	+0.80	–	–
X_2	4.91	1.10	–	–0.56	–
X_3	594	85	–	–	–0.40

where X_1 = Seed per acre, X_2 = rainfall in inches X_3 = Accumulated temperature above 42°F. (8)

Or

(b) (i) From the following data, find

(1) The two regression equations

(2) The coefficient of correlation between the marks in economics and statistics

(3) The most likely marks in statistics when marks in Economics are 30.

Marks in Economics :	25	28	35	32	31	36	29	38	34	32
Marks in Statistics :	43	46	49	41	36	32	31	30	33	39

(8)

(ii) A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$p(X)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

Find

(1) the value of K

(2) $P(1.5 < X < 4.5 / X > 2)$ and

(3) the smallest value of λ for which $P(X \le \lambda) > \frac{1}{2}$. (8)

14. (a) (i) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance. (8)

(ii) For the double Poisson distribution :

$$p(x) = p(X = x) = \frac{1}{2}\cdot\frac{e^{-m_1} m_1^x}{x!} + \frac{1}{2}\cdot\frac{e^{-m_2} m_2^x}{x!};\ x = 0,1,2,\cdots$$

Show that the estimates for m_1 and m_2 by the method of moments are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - \mu_1'^2}$. (8)

Or

(b) (i) Prove that the maximum likelihood estimate of the parameter α of a population having density function :

$$\frac{2}{\alpha^2}(\alpha - x), 0 < x < \alpha$$

for a sample of unit size is $2x$, x being the sample value. Show also that the estimate is biased. (8)

(ii) Let $x_1, x_2, x_3, \cdots, x_n$ be a random sample from the uniform distribution with p.d.f.

$$f(x,\theta) = \frac{1}{\theta}, 0 < x < \infty, \theta > 0$$
$$= 0, \text{ elsewhere}$$

Obtain the maximum likelihood estimator for θ. (8)

15. (a) Discuss Fuzzy algorithm in neural sets. (16)

Or

(b) Discuss Fuzzy genetic algorithms. (16)
