C 3281

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Fourth Semester

Biotechnology

MA 1255 – PROBABILITY AND STATISTICS

(Regulation 2004)

Time : Three hours | Maximum : 100 marks

Statistical tables are permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the moment generating function of a random variable X whose probability mass function is $P(X = x) = q^{x-1}p, x = 1, 2, 3, \ldots$.

2. Write the mean and variance of Weibull Distribution.

3. Find the marginal density function of Y if the joint probability density function of a two dimensional random variable (X, Y) is $f(x,y) = \begin{cases} 2, & 0 < x < 1, \quad 0 < y < x \\ 0, & \text{elsewhere} \end{cases}$.

4. Find the correlation coefficient between X and Y if the lines of regression are $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$.

5. Prove that the Poisson process is not a stationary process.

6. If the transition probability matrix of a Markov chain is $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, find the steady state probability distribution.

7. Find the hazard rate function of the exponential distribution with probability density function $f(t) = \lambda e^{-\lambda t}, \ t > 0$.

8. Classify the two types of maintenance.

9. What is the aim of the design of experiment?

10. When do you say that a process is out of control?

PART B — (5 × 16 = 80 marks)

11. (a) (i) A and B throw alternatively with a pair of ordinary dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chance of winning. (8)

(ii) The daily consumption of milk in a city in excess of 20,000 liters is approximately distributed as an Gamma variate with the parameters $k = 2$ and $\lambda = \frac{1}{10,000}$. The city has a daily stock of 30,000 liters. What is the probability the stock is insufficient on a particular day. (8)

Or

(b) (i) Find the moment generating function of binomial distribution and hence find its mean and variance. (8)

(ii) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes. (8)

12. (a) (i) If the joint distribution function of X and Y is given by $f(x, y) = \begin{bmatrix} 1 - e^{-x} - e^{-y} + e^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{elsewhere} \end{bmatrix}$. Find the marginal densities of X and Y. Also find whether X and Y are independent or not. (8)

(ii) Let $X_1, X_2, X_3, \ldots X_n$ be a identically Poisson variates with parameter λ. Use central limit theorem to estimate $P(120 \le S_n \le 160)$ where $S_n = X_1 + X_2 + X_3 + \cdots X_n$, $\lambda = 2$ and $n = 75$. (8)

Or

(b) (i) Find the equation of lines of regression from the following data.

X:	68	64	75	50	64	80	75	40	55	64
Y:	62	58	68	45	81	60	68	48	50	70

(8)

(ii) Let (X, Y) be a two-dimensional random variables having joint density function $f(x, y) = \left[4xye^{-(x^2+y^2)} : x \ge 0, y \ge 0\right]$, find the density function of $U = \sqrt{X^2 + Y^2}$. (8)

13. (a) (i) A stochastic process is described by $X(t) = A\sin t + B\cos t$ where A and B are independent random variables with zero mean and equal standard deviations. Show that the process is stationary of order two. (8)

(ii) A gambler's luck follows a pattern. If he wins a game, the probability of his winning the next game is 0.6. However if he loses a game, the probability of his losing the next game is 0.7. There is even chance that the gambler wins the first game. What is the probability that he wins the second game, the third game and in the long run? (8)

Or

(b) (i) Derive the differential equations of birth and death process. (8)

(ii) Patients arrive randomly and independently at a doctor's consulting room from 8 AM at an average rate of one in 5 minute. The waiting room can hold 12 persons. What is the probability that the room will be full when the doctor arrives at 9 AM? (8)

14. (a) (i) The density function of the time to failure of an appliance is $f(t) = \frac{32}{(t+4)^3}$, $t > 0$ is in years. Find the reliability function $R(t)$ and failure rate $\lambda(t)$. (8)

(ii) A new computer has a constant failure rate of 0.02 per day and a constant repair rate of 0.1 per day. Compute the interval availability for the first 30 days and the steady state availability. (8)

Or

(b) (i) There are 16 components in a non-redundant system. The average reliability of each component is 0.99. In order to achieve at least this system reliability by using a redundant system with 4 identical new components what should be the least reliability of each new component? (8)

(ii) If a device has a failure of $\lambda(t) = 0.015 + 0.02t$ per year where t is in years, calculate the reliability for a 5 year design life assuming that no maintenance is performed. Also find the reliability for a 5 year design life, assuming that annual preventive maintenance restores the device to an as good as new condition. (8)