

B 2315

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Fourth Semester

Chemical Engineering

MA 036 — STATISTICS AND LINEAR PROGRAMMING

(Common to Textile Technology, Textile Chemistry and Leather Technology)

Time : Three hours

Maximum : 100 marks

Statistical Tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the expectation of obtaining different numbers in rolling a die?
2. Show that $f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$, $x = 0, 1, 2, \dots, \infty$ is a pdf.
3. Write the two regression line equations.
4. Write the conditional density function of X given Y , where X and Y are the random variables.
5. What do you understand by tolerance limits?
6. Discuss briefly Latin Square design.
7. Write the canonical form of a Linear Programming Problem.
8. When do you make use of artificial variables in a Linear Programming Problem?
9. What are the different methods used in finding the initial basic feasible solution in a transportation problem?
10. Discuss briefly an unbalanced assignment problem.

PART B — (5 × 16 = 80 marks)

11. (a) (i) State and prove Chebychev inequality. (8)
- (ii) Obtain the moment generating function for a gamma distribution. Hence obtain its variance. (8)

Or

- (b) (i) If the life X in (years) of a certain type of car has a Weibull distribution with the parameter $\beta = 2$, find the value of the parameter α , given that the probability that the life of the car exceeds 5 years is $e^{-0.25}$ for these values of α and β , find the mean and variance of X . (8)
- (ii) Obtain the mean and variance of geometric distribution. (8)
12. (a) The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}$, $0 \leq x \leq 2$, $0 \leq y \leq 1$. Compute $P(X > 1)$; $P\left(Y < \frac{1}{2}\right)$, $P\left(X > 1/Y < \frac{1}{2}\right)$, $P\left(Y < \frac{1}{2}/X > 1\right)$, $P(X < Y)$ and $P(X + Y \leq 1)$. (16)

Or

- (b) (i) Prove the central limit theorem. (8)
- (ii) Given the joint density function of X and Y as

$$f(x, y) \begin{cases} = \frac{1}{2} x e^{-y}, & 0 < x < 2, y > 0 \\ = 0, & \text{elsewhere.} \end{cases}$$

Find the distribution function of $(X + Y)$. (8)

13. (a) The weights in gm of a number of copper wires, each of length 1 metre, were obtained. These are shown classified according to the dye from which they come : (16)

$$D_1 : 1.30, 1.32, 1.36, 1.35, 1.32, 1.37$$

$$D_2 : 1.28, 1.35, 1.33, 1.34$$

$$D_3 : 1.32, 1.29, 1.31, 1.28, 1.33, 1.30$$

$$D_4 : 1.31, 1.29, 1.33, 1.31, 1.32$$

$$D_5 : 1.30, 1.32, 1.30, 1.33.$$

Test the hypothesis that there is no difference between the mean weights of wire coming from different dyes. Use 5% level of significance.

Or

- (b) Analyse the variance in the following latin square by using 1% level of significance. (16)

| | | | |
|---------|---------|---------|---------|
| 20 B | 17 C | 25 D | 34 A |
| 23 A | 21 D | 15 C | 24 B |
| 24 D | 26 A | 21 B | 19 C |
| 26 C | 23 B | 27 A | 22 D |

14. (a) Solve the Linear Programming problem given below : (16)

$$\text{Maximize } Z = 5x_1 + 12x_2 + 4x_3$$

subject to

$$x_1 + 2x_2 + x_3 \leq 10$$

$$2x_1 - x_2 + 3x_3 = 8$$

$$x_1, x_2, x_3 \geq 0.$$

Or

- (b) Obtain the solution of the following Linear Programming Problem using simplex method. (16)

$$\text{Minimize } Z = x_1 - 3x_2 - 2x_3$$

subject to

$$\begin{aligned} 3x_1 - x_2 + 2x_3 &\leq 7 \\ -2x_1 + 4x_2 &\leq 12 \\ -4x_1 + 3x_2 + 8x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

15. (a) Obtain the starting solution in the following transportation problem by Vogel's approximation method. Also obtain the optimal solution. (16)

| | | | | |
|----|----|----|----|----|
| 10 | 20 | 5 | 7 | 10 |
| 13 | 9 | 12 | 8 | 20 |
| 4 | 15 | 7 | 9 | 30 |
| 14 | 7 | 1 | 0 | 40 |
| 3 | 12 | 5 | 19 | 50 |
| 60 | 60 | 20 | 10 | |

Or

- (b) Solve the following assignment problem using Hungarian technique the matrix entries are presenting times in hours. (16)

| | | Operators | | | | |
|------|---|-----------|----|----|----|----|
| Jobs | 1 | 16 | 19 | 21 | 19 | 25 |
| | 2 | 23 | 14 | 17 | 20 | 20 |
| | 3 | 4 | 26 | 24 | 24 | 8 |
| | 4 | 15 | 18 | 16 | 15 | 17 |
| | 5 | 20 | 21 | 34 | 22 | 19 |