

B 2316

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Fifth semester

Chemical Engineering

MA 037 — SPECIAL FUNCTIONS, DIFFERENCE EQUATION AND
Z-TRANSFORMS

(Common to Leather Technology and Textile Technology)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define Gamma function.
2. Define ordinary point of second order linear ordinary differential equation.
3. Write down the solution of the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - 9/4)y = 0 \text{ in terms of Bessel functions.}$$

4. Define modified Bessel function.
5. Express $2x - 5$ in terms of Legendre polynomials.
6. Prove that $P_n(-1) = (-1)^n$
7. Write down the generating function of the Hermite's polynomial.
8. State Rodrigue's formula for Laguerre polynomial.
9. Form the differences equation by eliminating "a" from $y_n = a3^n$.
10. Find the Z-transform of the sequence $\{a^n\}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $\beta(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$ and evaluate

$$\int_0^1 \frac{x^6}{\sqrt{1-x^2}} dx. \quad (8)$$

- (ii) Prove that $\frac{\beta(m+1, n)}{m} = \frac{\beta(m, n+1)}{n} = \frac{\beta(m, n)}{m+n}$. (8)

Or

- (b) Find the general solution in series of ascending powers of x , of the differential equation $4x \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = 0$. (16)

12. (a) Prove that $e^{x/2(t-1/t)}$ is the generating function of Bessel functions.

Or

- (b) (i) Prove that $J_{-1/2}(x) = \left(\sqrt{\frac{2}{\pi x}} \right) \cos x$. (8)

- (ii) Prove that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$. (8)

13. (a) (i) Prove that $(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0$. (8)

- (ii) Derive the Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n (x^2-1)^n}{dx^n}$. (8)

Or

- (b) Prove that $\int_{-1}^1 P_m(x)P_n(x)dx = 0$ if $m \neq n$
 $= \frac{2}{2n+1}$ if $m = n$ (16)

14. (a) (i) Show that $H'_n(x) = 2nH_{n-1}(x)$. (8)

- (ii) Prove that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$. (8)

Or

- (b) Show that $\int_0^a e^{-x} L_m(x)L_n(x)dx = 0$ if $m \neq n$ (16)
 $= (n!)^2$ if $m = n$

15. (a) (i) Solve the difference equation $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$ given that $y_0 = y_1 = 0$. (8)

(ii) Find the Z-transform of $r^n \cos n\theta$ and $r^n \sin n\theta$. (8)

Or

(b) (i) Find the inverse transforms of

$$\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)} \text{ and } \frac{4z^3}{(2z-1)^2(z-1)}. \quad (4+4)$$

(ii) Using Z-transform, solve the difference equation $y_{n+2} + 4y_{n+1} + 4y_n = n$ given that $y_0 = 0$ and $y_1 = 1$. (8)