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M.C.A. DEGREE EXAMINATION, MAY/JUNE 2007.

Elective

MC 1621 — NUMERICAL AND STATISTICAL METHODS

(Regulation 2005)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define an ill-conditioned system of equations and give an example.

2. State the Gauss-Seidal iteration formula to solve a system of three linear equations having iteration index $(n + 1)$.

3. State the Newton's forward difference interpolation formula for equal intervals.

4. State the differentiation formula for the second order derivative of a tabulated data for equal intervals with forward differences

5. State the Euler's modified formula to solve $y' = f(x, y), y(x_0) = y_0$.

6. What is the total error with respect to differential equation and state its two components?

7. State the axioms of probability.

8. Define conditional probability distribution of a random variable.

9. List the assumptions on the data to apply Student's t-test.

10. State the χ^2– statistic for a 2×2 contingency table after carrying out Yate's correction.

PART B — (5 × 16 = 80 marks)

11. (a) Solve the following equations by Gauss—Jordan elimination method :

$x + y + z + 4w = -6$; $x + 7y + z + w = 12$; $x + y + 6z + w = -5$; $5x + y + z + w = 4$.

Or

(b) Solve by Gauss-Seidal iterative method, the following equations :

$x + y + 54z = 110$; $27x + 6y - z = 85$; $6x + 15y + 2z = 72$.

12. (a) (i) From the data given below, find the number of students whose weight is between 60 and 70 lbs.

Weight in lbs. (x) :	20-40	40-60	60-80	80-100	100-120
No. of students (y) :	250	120	100	70	50

(ii) Use Lagrange's interpolation formula to find $f(6)$, given the data :

X	2	5	7	10	12
$f(x)$	18	180	448	1210	2028

Or

(b) (i) Find the first and second derivative of the function tabulated as below at $x = 0.6$:

x :	0.4	0.5	0.6	0.7	0.8
y :	1.5836	1.7974	2.0442	2.3275	2.6511

(ii) A curve passes through the points (1, 2), (1.5, 2.4), (2, 2.7), (2.5,2.8), (3, 3), (3.5, 2.6) and (4, 2.1). Find the area bounded by the curve the x - axis, $x = 1$ and $x = 4$. Also, find the volume of solid of revolution got by revolving this area about x-axis.

13. (a) (i) Use Tailor series method to find y at $x = 0.1, 0.2$ given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$. (correct to four decimal places).

(ii) Apply fourth order Runge-Kutta method and find $y (0.7)$ correct to 4 decimal places if $\frac{dy}{dx} = y - x^2$, $y(0.6) = 1.7379$.

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(b) Find $y(0.1)$, $y(0.2)$, $y(0.3)$ by Runge-Kutta method of order 4 and find $y(0.4)$ by Adam's method for the equation $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$.

14. (a) (i) State and prove the Baye's theorem on probability. (10)

(ii) A random variable X has the density function

$$f(x) = \begin{cases} \frac{1}{4}, & -2 < x < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the values of $P(X < 1)$, $P(|X| > 1)$ and $P(2X + 3 > 5)$. (6)

Or

(b) (i) Obtain the moment generating function of the random variable X having probability density function

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

(ii) Joint pdf of two-dimensional random variable (X, Y) is given by $f(x, y) = 4xy \ \exp(-(x^2 + y^2))$, $x, y \ge 0$. Test whether X and Y are independent. For the above pdf, find the conditional density of X given $Y = y$.

15. (a) (i) Memory capacity of 9 students was tested before and after a course on meditation for a month. State whether the course was effective or not from the data given below:

Before: 10 15 9 3 7 12 16 17 4

After: 12 17 8 5 6 11 18 20 3

(ii) Two independent samples of sizes 9 and 7 from a normal population had the following values:

Sample I: 18 13 12 15 12 14 16 14 15

Sample II: 16 19 13 16 18 13 15

Do the estimates of the population variance differ significantly at 5% level?

Or

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(b) As part of the investigation of the collapse of the roof of a building, a testing laboratory is given all the available bolts that connected the steal structure at three different position on the roof. The forces required to shear each of these bolts are as follows:

Position I :   90 82 79 98 83 91

Position II :   105 89 93 104 89 95 86

Position III :   83 89 80 94

Perform an ANOVA to test at 5% level of significance whether the differences among the sample means at the three positions are significant.