G 3529

M.C.A. DEGREE EXAMINATION, MAY/JUNE 2007.

Second Semester

MC 1651 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulation 2005)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the eigen values of A^2 where $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$.

2. Find the rank of the matrix $\begin{pmatrix} 3 & -2 & 0 \\ 0 & -2 & -3 \\ 0 & 1 & 2 \end{pmatrix}$.

3. Given $A = \{2,5,6\}$, $B = (3, 4, 2\}$ $C = \{1, 3, 4\}$ find $A - B$ and $A - C$.

4. Give an example of a function which is one–one and not onto.

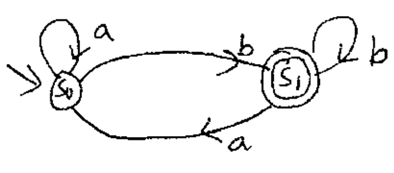
5. Determine whether the conclusion q follows logically from the premises $H_1 : p \rightarrow q$ $H_2 : p$.

6. Express the following statement in symbolic form "Any integer is either positive or negative".

7. Define a phrase structure grammar.

8. Give a context free language to generate a palindrome in $\Sigma = \{a, b\}$.

9. In a finite automata.



Find whether the following are accepted (a) $a^m b^n$ (b) $a^n b^m a^k$.

Define a regular grammar.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (10)

(ii) Determine the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. (6)

Or

(b) Show that the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ satisfies its own characteristic equation. Hence find A^{-1}. (16)

12. (a) (i) Let $X = \{1, 2, 3, 4, 5\}$ and $R = \{\langle x\ y\rangle / x - y$ is divisible by 3$\}$. Find whether R an equivalance relation, also find the graph of R. (8)

(ii) Show that for any three sets A, B, C (8)

$A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Or

(b) (i) Let $X = \{2, 3, 4, 6, 12, 36, 48\}$ and let R be the relation xRy if x divides y. Draw the Hasse diagram of R. (8)

(ii) f, g, h are functions from Z into Z defined by $f(x) = x + 5$ $g(x) = x - 2$, $h(x) = x^2$. Define (1) fg (2) f^3 (3) fh. (8)

13. (a) (i) Obtain the *pcnf* and *pdnf* of $(\neg p \to R) \wedge (q \rightleftharpoons p)$. (8)

(ii) Prove that $(p \wedge q) \Rightarrow (p \to q)$. (8)

Or

(b) (i) Find the consistency of the following premises $p \to q, p \to r$, $q \to \neg r, p$ (8)

(ii) Prove that $(\exists x)\ ((p(x) \wedge q(x))) \Rightarrow (\exists x) p(x) \wedge (\exists x)\ (q(x))$. (8)

14. (a) (i) Explain the four classes of grammars with an example. (8)

(ii) Find the language generated by the grammar : (8)

$S \to 0SBA / 01A,\ AB \to BA,\ 1B \to 11,\ 1A \to 10,\ 0A \to 00.$

Or

(b) (i) Construct a suitable grammar for $L = \{a^n b^{2n} / n \geq 1\}$. (8)

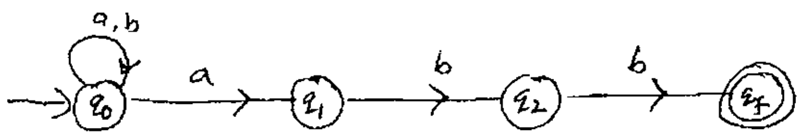
(ii) Prove that $L = \{a^n b^n : n \geq 1\}$ is not regular. (8)

15. (a) (i) Construct a finite state automata that accepts those strings over $\{a, b\}$ that contain aaa as substring. (8)

(ii) Prove that for every NFA there exist an equivalent DFA. (8)

Or

(b) (i) Consider a NFA as shown below, find equivalent DFA. (8)



(ii) Prove that for every regular expression(r) there exists an finite automata to accept r. (8)