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P 1305

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Fourth Semester

Electronics and Communication Engineering

MA 034 — RANDOM PROCESSES

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Let A_1 and A_2 be events such that $P(A_1) = 0.6, P(A_2) = 0.4$ and $P(A_1 \cup A_2) = 0.8$. Are A_1 and A_2 independent?

2. A discrete random variable X has moment generating function $M_X(t) = e^{2(e^t - 1)}$. Find $E(X), Var(X)$ and $P(X = 0)$.

3. Let X and Y be two discrete random variables with joint probability mass function $P(X = x, Y = y) = \begin{cases} \dfrac{x + 2y}{18}, & x = 1, 2, \quad y = 1, 2 \\ 0, & \text{otherwise.} \end{cases}$

   Find the marginal probability mass function of X and $E(X)$.

4. State the central limit theorem for independent identically distributed random variables.

5. Define Binomial process and state its properties.

6. Let $X(t)$ be a Poisson process with rate λ. Find $E(X(t)\,X(t + \tau))$, where $\tau > 0$.

7. The random process $X(t)$ has an autocorrelation function $R_{XX}(\tau) = 18 + \dfrac{2}{6 + \tau^2}[1 + 4\cos(12\tau)]$. Find $E(X(t))$ and $E(X^2(t))$.

8. If $R_{XX}(\tau)$ is the autocorrelation function of a stationary process, prove that it is an even function of τ.

9. A wide-sense stationary noise process $N(t)$ has an autocorrelation function $R_{NN}(\tau) = Pe^{-3|\tau|}, -\infty < \tau < \infty$, with P as a constant. Find its power density spectrum.

10. State and prove any one of the properties of cross-spectral density function.

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) In each of the following cases $M_X(t)$ the moment generating function of X is given. Determine the distribution of X and its mean.

(1) $M_X(t) = \left(\frac{1}{4}e^t + 3/4\right)^7$

(2) $M_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^n$

(ii) If a random variable X has geometric distribution, i.e. $P(X = x) = pq^{x-1}, x = 1, 2, 3, \cdots$, where $q = 1 - p$ and $0 < p < 1$, show that $P(X > x + y / X > y) = P(X > x)$.

(iii) The average IQ score on a certain campus is 110. If the variance of these scores is 15, what can be said about the percentage of students with an IQ above 140?

Or

(b) (i) If X is a continuous random variable with p.d.f.

$$f(x) = \begin{cases} x, \ 0 \le x < 1 \\ 3/2\,(x-1)^2, 1 \le x < 2 \\ 0, \text{otherwise,} \end{cases}$$

find the cumulative distribution function $F(x)$ of X.

(ii) If X is a continuous random variable having the p.d.f.

$$f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise,} \end{cases}$$

and $Y = e^{-X}$, find the p.d.f. of the random variable Y.

(iii) Find the moment generating function $M_X(t)$ of the R.V. X having the p.d.f.

$$f(x) = \begin{cases} \frac{1}{2}x^2 \, e^{-x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find $E(X)$ also.

12. (a) (i) Find $P(X > 2/Y < 4)$ when the joint p.d.f. of X and Y is given by

$$g(x, y) = \begin{cases} e^{-(x+y)}, & x \ge 0, y \ge 0 \\ 0, & \text{otherwise.} \end{cases}$$

(ii) Let X and Y be non-negative continuous random variables having the joint probability density function

$$f(x, y) = \begin{cases} 4xy \, e^{-(x^2+y^2)}, & x > 0, y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find the p.d.f. of $U = \sqrt{X^2 + Y^2}$.

Or

(b) (i) If the joint p.d.f. of X and Y is given by

$$g(x, y) = \begin{cases} e^{-(x+y)}, & x \ge 0, y \ge 0 \\ 0, & \text{otherwise,} \end{cases}$$

then

(1) find the m.p.d.f. of X

(2) find the m.p.d.f. of Y

(3) Are X and Y independent RVs? Explain.

(4) Find $P(X > 2, Y < 4)$

(5) Find $P(X > Y)$.

(ii) If $f(x, y) = \begin{cases} 2 - x - y & \text{for } 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{elsewhere,} \end{cases}$

is the joint p.d.f. of the random variables X and Y, find the correlation coefficient of X and Y.

13. (a) (i) Show that the random process $\{X(t)\}$ where $X(t) = A\cos(wt + \theta)$ with A and w are constants and θ is a uniform random variable over $(-\pi, \pi)$, is wide sense stationary (WSS)

(ii) Suppose $X(t)$ is a random telegraph type process composed of pulses of heights +1 and −1 respectively. The number of transitions of the t axis in a time 2 is given by $P(k \text{ transitions}) = \frac{4^k e^{-4}}{\underline{|k}}$.

(1) Classify the above process (2) Whether $X(t)$ is ergodic? Explain.

Or

(b) (i) Prove that the difference of two Poisson processes is not a Poisson process.

(ii) Show that the random process $X(t) = \cos(t + \phi)$ where ϕ is uniformly distributed over $(0, 2\pi)$ is

(1) First order stationary

(2) Stationary in the wide sense.

14. (a) Given the random process $X(t) = \sum_{n=-\alpha_1}^{\alpha_1} A_n\, g(t - nb + \phi)$ where

$$g(t) = \begin{cases} 1, & 0 < t < b \\ 0, & \text{otherwise.} \end{cases}$$

Here ϕ and A_n's are independent random variables with density functions $\delta_o(\alpha) = \frac{1}{b}, 0 < \alpha < b$ and $\delta_{A_n}(\alpha) = \frac{1}{2}\delta(\alpha - 1) + \frac{1}{2}\delta(\alpha + 1)$ and the joint probability mass function for the A_n's is given by

A_n \ A_{n+1}	-1	$+1$
-1	1/4	1/4
$+1$	1/4	1/4

Find the autocorrelation function $R_{XX}(\tau)$ in $0 < \tau < b$.

Or

(b) Given $X(n) = \sum_k X_k \delta(n-k)$ where the joint probability mass function for the random variables is given by

X_k \ X_{k+1}	0	1
0	0.2	0.3
1	0.3	0.2

Find the autocorrelation function $R_{XX}(\tau)$ for $k = 0, 1, 2, 3$ and covariance function $C_{XX}(k)$ for $k = 0, 1, 2, 3$.

15. (a) (i) If the power spectral density of a WSS process is given by

$$S(W) = \begin{cases} \frac{b}{a}(a - |w|), & |w| \leq a \\ 0, & |w| > a. \end{cases}$$

Find the autocorrelation function of the process.

(ii) Show that in an input-out system the energy of a signal is equal to the energy of its spectrum.

Or

(b) (i) The auto correlation function of a signal is $R_{XX}(\tau) = e^{-\frac{\tau^2}{2k^2}}$, where k is a constant. Find the power spectral density and average power.

(ii) A wide sense stationary random process $X(t)$ with autocorrelation function $R_{XX}(\tau) = A\, e^{-\alpha|\tau|}$ where A and α are real positive constants, is applied to the input of an linearly time invariant system with impulse response $h(t) = e^{-bt}, t > 0$, where b is a real positive constant. Find the spectral density of the output $Y(t)$ of the system.
