Reg. No. : | | | | | | | | | | | |

P 1306

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Fourth Semester

Chemical Engineering

MA 036 - STATISTICS AND LINEAR PROGRAMMING

(Common to Textile Technology, Textile Chemistry and Leather Technology)

Time : Three hours — Maximum : 100 marks

Statistical Tables may be permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A continuous random variable X has a probability density function $f(x) = k(1+x), 2 \leq x \leq 5$. Find k and $P(X < 4)$.

2. Define Gamma distribution.

3. The probability density function of a bivariate random variable (X, Y) is given by

$$f(x, y) = \begin{cases} \frac{1}{8}(x+y), 0 < x < 2, 0 < y < 2 \\ 0, \text{otherwise.} \end{cases}$$

   Find the marginal probability density function of X.

4. State the central limit theorem.

5. Define completely randomized design.

6. Find the lower and upper control limits for p-chart when $n = 200$ and $\overline{P} = 0.06$.

7. Write down the general linear programming problem in Matrix form.

8. What is meant by an artificial variable?

9. What is meant by degenerate solution in a Transportation problem?

10. State the fundamental theorem of Duality.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the moment generating function of the Poisson random variable with parameter λ and hence find its mean and variance. (8)

(ii) The density function of a random variable X is given by

$$f(x) = \begin{cases} \frac{x}{6}, x = 1, 2, 3 \\ 0, \text{ otherwise.} \end{cases}$$

Find $E(X)$, $Var\ (X)$, $E(X^3)$ and $E(X^3 + 2X + 7)$. (8)

Or

(b) (i) State and prove the memoryless property of Geometric distribution. (8)

(ii) If the probability that an applicant for a driver's license will pass the road test an any given trial is 0.8, what is the probability that he will finally pass the test (1) on the fourth trial and (2) in fewer than 4 trials? (8)

12. (a) (i) The random variables X and Y have the joint probability density function

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional probability density function of X given Y and Y given X. (8)

(ii) The joint density function of random variables X and Y is given by

$$f(x, y) = \begin{cases} x + y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the correlation co-efficient between X and Y. (8)

Or

(b) (i) Let X_1 and X_2 be two continuous random variables with joint probability distribution

$$f(x_1, x_2) = \begin{cases} 4x_1x_2, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the joint probability distribution of X_1^2 and X_1X_2.

(ii) Given the joint probability density function of (X, Y) is

$$f(x, y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find whether X and Y are independent. Also find $P[X + Y > 1]$. (8)

13. (a) (i) Three varieties of a crop are tested in a randomized block design with four replication, the layout is given below. The yields are given in kilograms.

C 48	A 51	B 52	A 49
A 47	B 49	C 52	C 51
B 49	C 53	A 49	B 50

Perform a randomised block analysis of variance. Use a 0.05 level of significance. (10)

(ii) The following data gives the average life in hours and range in hours of 12 samples each of 5 lamps. Construct the control chart for $\overline{X}$ and R and comments on the state of control. (6)

$\overline{X}$:	120	127	152	157	160	134	137	123	140	144	120	127
R :	30	44	60	34	38	35	45	62	29	50	35	41

Or

(b) (i) Analyse the following results of a Latin square experiment :

Row/Column	1	2	3	4
1	A (12)	D (20)	C (16)	B (10)
2	D (18)	A (14)	B (11)	C (14)
3	B (12)	C (15)	D (19)	A (13)
4	C (16)	B (11)	A (15)	D (20)

Here the letters A, B, C, D denote the treatments and the figures in brackets denote the observations. (12)

(ii) Explain the basic principles of experimental design. (4)

14. (a) Solve the following LPP to show that it has alternative optimal solutions (16)

Maximize $z = x_1 + 2x_2 + 3x_3$

subject to the constraints

$$2x_1 + x_2 + 5x_3 \geq 20$$
$$x_1 + 2x_2 + 3x_3 + x_4 \geq 10$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

Or

(b) Solve the following linear programming problem by using the Big-M method (16)

Maximize $z = 4x_1 + 2x_2$

subject to the constraints

$$3x_1 + x_2 \geq 27$$
$$-x_1 + x_2 \leq 21$$
$$x_1 + 2x_2 \geq 30$$
$$x_1, x_2 \geq 0.$$

15. (a) Apply the principle of duality to solve the following LPP : (16)

Min. $z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

subject to the constraints

$$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$$
$$x_2 + 5x_3 - 6x_4 \geq 10$$
$$2x_1 + 5x_2 + x_3 + x_4 \geq 8$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

Or

(b) Solve the following problem : (16)

	1	2	3	4	Supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	