Reg. No. :

Q 2373

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Fifth Semester

Chemical Engineering

MA 037 — SPECIAL FUNCTIONS, DIFFERENCE EQUATIONS AND Z-TRANSFORMS

(Common to Leather Technology and Textile Technology)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^\infty e^{-x^2} x^4 dx$.

2. Find $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx$.

3. Define Bessel function by its generating function.

4. Give the differential equation whose one solution is $J_{-1/2}(x)$.

5. Express $x^2 - 5x$ in terms of Legendre polynomials

6. Show that all the roots of $P_n(x) = 0$ are real.

7. Show that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.

8. State the orthogonal property of Leguerre polynomials.

9. Find the Z-transform of n^2.

10. Form the difference equation by eliminating k from $y_n = k\,4^n$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\int_0^1 \left(x \log \frac{1}{x}\right)^{1/3} dx$. (8)

(ii) Evaluate $\int_0^\infty x^p e^{-ax^q} dx$ where a, p and q are positive constants in terms of Gamma function. (8)

Or

(b) Solve $\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n\,(n+1)\,y = 0$. (16)

12. (a) (i) Show that $\frac{d}{dx}\left(x^n J_n\,(x)\right) = x^n\; J_{n-1}(x)$. (8)

(ii) Show that $\cos x = J_0 - 2J_2 + 2J_4$. (8)

Or

(b) (i) Show that $J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin n\theta)\,d\theta$. (8)

(ii) Show that $J_0^2 + 2\left(J_1^2 + J_2^2 + \cdots\right) = 1$. (8)

13. (a) (i) Show that $P_n(x)$ is the coefficient of h^n in the expansion of $\left(1-2xh+h^2\right)^{-1/2}$ in ascending powers of x. (8)

(ii) Prove that $nP_n\,(x) = x\frac{d}{dx}\,P_n(x) - \frac{d}{dx}P_{n-1}(x)$. (8)

Or

(b) (i) Show that $P_n(1) = 1$, $P_n(-x) = (-1)^n P_n(x)$ and $P_n(-1) = (-1)^n$. (8)

(ii) Show that $\int_{-1}^{+1} P_n(x)\,dx = 0, (n \neq 0)$. Find $\int_{-1}^{+1} P_0(x)\,dx$. (8)

14. (a) Solve Hermite differential equation. (16)

Or

(b) (i) Show that $\frac{1}{1-t}\,e^{-xt/1-t} = \sum_{n=0}^{\infty} L_n(x)\,t^n$. (8)

(ii) Find expression for $H_0\,(x), H_1\,(x)$ and $H_2(x)$. (8)

15. (a) (i) Solve $y_{n+2} - 6y_{n+1} + 5y_n = 2^n$ with $y_0 = y_1 = 0$. (8)

(ii) Using convolution theorem find the inverse Z-transform of $\left(\frac{z}{z-a}\right)^2$. (8)

Or

(b) (i) Find the inverse Z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$. (8)

(ii) Using Z-transform, Solve $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = y_1 = 0$. (8)