Reg. No. : | | | | | | | | | | |

**P 1307**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Fourth Semester

Aeronautical Engineering

MA 038 — NUMERICAL METHODS

(Common to : Automobile Engineering, Civil Engineering, Instrumentation Engineering, Instrumentation and Control Engineering, Mechanical Engineering, Mechatronics Engineering and Production Engineering)

Time : Three hours                                        Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. By Gauss elimination method solve

   $11x + 3y = 17$

   $2x + 7y = 16.$

2. State the condition for convergence of Jacobi's iteration method for solving a system of simultaneous algebraic equations.

3. Form the divided difference table for

| | | | | |
|---|---|---|---|---|
| $x$ : | 1 | 3 | 6 | 11 |
| $y$ : | 4 | 32 | 224 | 1344 |

4. State Gauss backward difference interpolation formula.

5. If $I_1 = 0.775$, $I_2 = 0.7828$ find I using Romberg's method.

6. Using Simpson's rule find $\int_0^4 e^x \, dx$ given $e^0 = 1, e^1 = 2.72, e^2 = 7.39, e^3 = 20.09$ and $e^4 = 54.6$.

7. Find $y(0.1)$ given $\frac{dy}{dx} = \frac{1}{2}(x + y), y(0) = 1$ by modified Euler's method.

8. Write down the Milne's predictor and corrector algorithms.

9. Write down the finite difference form of the equation $\nabla^2 u = f(x, y)$.

10. Write an explicit formula to solve the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the root of the equation $xe^x = \cos x$ using the regula-falsi method correct to four decimal places. (8)

(ii) Find the numerically largest eigenvalue of $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and the corresponding eigenvector. (8)

Or

(b) (i) Determine the roots of equations

$x^2 + xy = 6, x^2 - y^2 = 3$

using the Newton–Raphson method, starting with initial approximation $(1, 1)$. (Calculate the result up to second iteration). (8)

(ii) Using Gauss–Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. (8)

12. (a) (i) By the use of Lagrange's formula find the polynomial of degree three which takes the values prescribed below : (8)

| | | | | |
|---|---|---|---|---|
| $x$ : | 0 | 1 | 2 | 5 |
| $f(x)$ : | 2 | 3 | 12 | 147 |

(ii) Use Stirling's formula to find $y$ for $x = 35$ from the following table : (8)

| | | | | |
|---|---|---|---|---|
| $x$ : | 20 | 30 | 40 | 50 |
| $y$ : | 512 | 439 | 346 | 243 |

Or

(b) (i) Using Newtons forward interpolation formula, and the given table of values :

| | | | | | |
|---|---|---|---|---|---|
| $x$ : | 1.1 | 1.3 | 1.5 | 1.7 | 1.9 |
| $f(x)$ : | 0.21 | 0.69 | 1.25 | 1.89 | 2.61 |

Obtain the value of $f(x)$ when $x = 1.4$. (8)

(ii) Fit a polynomial to the data : (8)

| | | | | |
|---|---|---|---|---|
| $x$ : | −1 | 0 | 1 | 2 |
| $f$ : | −2 | 1 | 2 | 4 |

13. (a) (i) From the following table of values of $x$ and $y$, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$ : (8)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ : | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 |
| $y$ : | 2.7183 | 3.3201 | 4.0552 | 4.9530 | 6.0496 | 7.3891 | 9.0250 |

(ii) Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{1}{x+y}\,dx\,dy$ by Trapezoidal rule. (8)

Or

(b) (i) Evaluate $I = \int_0^1 \frac{dt}{1+t}$ by Gaussian formula with two points and three points and compare with the exact value. (8)

(ii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$. Hence determine the value of $\pi$. (8)

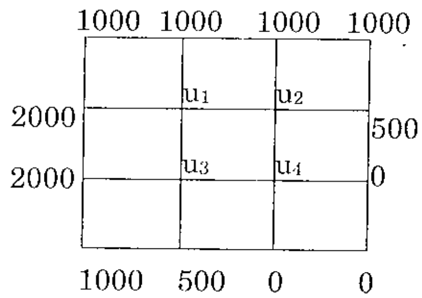
14. (a) (i) Compute $y(0.1)$, $y(0.2)$ and $y(0.3)$ if $y'' = y + xy'$, $y(0) = 1$, $y'(0) = 0$ by Taylor's series method. (8)

(ii) Using Adam's Bash forth method find $y(4.4)$ given $5xy' + y^2 = 2$, $y(4.0) = 1.0$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$. (8)

Or

(b) Given $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, $y(0) = 1$, $y'(0) = 0$, find the value of $y(0.1)$ by using Runge-Kutta method of fourth order. (16)

15. (a) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown : (16)




Or

(b) Derive the Bender Schmidt scheme of solving parabolic equation. Hence solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to $u(0, t) = 0$, $u(1, t) = 0$ and $u(x, 0) = \sin \pi x$, $0 < x < 1$. (16)