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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Fourth Semester

Metallurgical Engineering

MA 039 — PROBABILITY AND STATISTICS

(Common to Industrial Biotechnology)

Time : Three hours | Maximum : 100 marks

Statistical tables are permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Among the digits 1, 2, 3, 4, 5 at first one is chosen and then a second selection is made among the remaining four digits. Assuming that all 20 possible outcome have equal probabilities, find the probability that an odd digit will be selected in the first time and in the second time.

2. Define poisson distribution and find its mean.

3. Find k, if the joint probability density function of a bivariate random variable (X,Y) is given by $f(x,y) = \begin{cases} k(1-x)(1-y), & 0 < x, y < 1 \\ 0, & otherwise \end{cases}$

4. If the joint p.d.f. of (X,Y) is given by $f(x,y) = 24y(1-x), 0 \le y \le x \le 1$, find E(XY).

5. Define wide sense stationary and strict sense stationary random processes.

6. Show that the sum of two independent poisson process is a poisson process.

7. If a device has a failure rate of $\lambda(t) = (0.015 + 0.02t)$ per year, where t is in years. Then calculate the reliability for a 5 year design life assuming that no maintenance is performed.

8. The time to failure in operating hours of a critical solid - state power unit has the hazard rate function $\lambda(t)=0.003\,(\frac{t}{500})^{0.5}$, for $t \geq 0$. What is the reliability if the power unit must operate continuously for 50 hours?

9. What is meant by tolerance limits?

10. Compare RBD and LSD?

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has the following probability function. (8)

x :	0	1	2	3	4	5	6	7
p(x) :	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(1) Find k (2) Evaluate $P(X<6)$ (3) $P(X \geq 6)$ (4) $P(0<X<5)$.

(ii) The time required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. (time in hours)

(1) What is the probability that the repair time exceeds 2 hours?

(2) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours? (8)

Or

(b) (i) The diameter, say X, of an electric cable, is assumed to be a continuous random variable with p.d.f. $f(x)=6x(1-x)$, $0 \leq x \leq 1$.

(1) Check the above is a p.d.f. (2) Obtain expression for the c.d.f. (3) Compute $P(X \leq 1/2 / 1/3 \leq X \leq 2/3)$. (8)

(ii) Let the random variable X have the p.d.f.

$$f(x) = \begin{cases} \frac{1}{2}e^{-x/2}, x>0 \\ 0, \; otherwise \end{cases}$$

Find the mean, variance and moment generating function of X. (8)

12. (a) (i) The random variables X and Y have the joint p.d.f.

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, \; 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 \;\; otherwise \end{cases}$$

(1) Find the conditional p.d.f. of X given Y

(2) Are X and Y are independent? (8)

(ii) The number of computers sold daily at a shop is uniformly distributed with a minimum of 2000 and maximum of 5000 computers. Find (1) the probability that daily sales will fall between 2500 and 3000 computers. (2) what is the probability that the shop will sell atleast 4000 computers? (3) what is the probability that the shop will sell exactly 2500 computers? (8)

Or

(b) (i) If X and Y are independent random variables with p.d.f. $e^{-x}, x \geq 0$ *and* $e^{-y}, y \geq 0$, respectively, find the density function of $U = \frac{X}{X+Y}$ *and* $V = X + Y$. Are they independent? (8)

(ii) The joint p.d.f. of two random variables X and Y is given by

$$f(x,y) = \frac{9(1+x+y)}{2(1+x)^4 (1+y)^4}, 0 \leq x < \infty, 0 \leq y < \infty.$$

Find the marginal distributions of X and Y, and the conditional distribution of Y for $X = x$. (8)

13. (a) (i) If the WSS process {X(t)} is given by X(t) = 10 cos (100t + θ), where θ is uniformly distributed over $(-\pi, \pi)$, prove that {x(t)} is correlation ergodic. (8)

(ii) If $\{N_1(t)\}$ and $\{N_2(t)\}$ are poisson process with parameter λ_1 and λ_2 respectively. Show that $P\{N_1(t)=k / N_1(t)+N_2(t)=n\} = {}^nC_k \, p^k q^{n-k}$, where $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ and $q = \frac{\lambda_2}{\lambda_1 + \lambda_2}$. (8)

Or

(b) (i) The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3,...$ having 3 states 1,2 and 3 is

$$P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$ (8)

and the initial distribution is $p^{(o)} = (0.7, 0.2, 0.1)$. Find (1) $P(X_2 = 3)$ and (2) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

(ii) If the customers arrive in accordance with the poisson process, with mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 minute (2) between 1 and 2 minute (3) less than 4 minutes. (8)

14. (a) (i) Discuss the reliability under preventive maintenance and derive MTTF of a system with preventive maintenance. (8)

(ii) A system has 4 identical components connected in parallel and shows a system reliability of 0.90. How many more components should be added in parallel to get a system reliability of 0.99? (8)

Or

(b) (i) The density function of the time to failure (in years) of a component manufactured by a certain company is given by $f(t)=\dfrac{200}{(t+10)^3}, t \geq 0$.

(1) Derive the reliability function and determine the reliability for the first year of operation.

(2) Compute the MTTF.

(3) What is the design life for the reliability of 0.95? (8)

(ii) A critical communication relay has a constant failure rate of 0.1 per day. Once it has failed the mean time to repair is 2.5 days. What are the point availability at the end of 2 days, the interval availability over a 2 day period and the steady state availability? (8)

15. (a) Three varieties of crop are tested in a randomed block design with four replications, the layout being as given below : The yields are given in kilograms. Analyse for significance. (16)

C 48	A 51	B 52	A 49
A 47	B 49	C 52	C 51
B 49	C 53	A 49	B 50

Or

(b) Four doctors each test four treatments for a certain disease and observe the number of days each patient takes to recover. The results are as follows (recovery time in days). (16)

	Treatment			
Doctor	1	2	3	4
A	10	14	19	20
B	11	15	17	21
C	9	12	16	19
D	8	13	17	20

Discuss the difference between (i) doctors and (ii) treatments.