Reg. No. : | | | | | | | | | | | |

Q 2376

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Second Semester

Information Technology

MA 039 — PROBABILITY AND STATISTICS

Time : Three hours Maximum : 100 marks

Use of Statistical Table and Control Chart is permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If A and B are two events such that $P(A) = 0.4$, $P(A/B) = 0.6$ and $P(B/A) = 0.3$, find $P(B)$.

2. If X is a continuous random variable with p.d.f.

$$f(x) = \begin{cases} \frac{x}{6} + k, & 0 \le x \le 3 \\ 0, & \text{otherwise,} \end{cases}$$

then find k.

3. If the joint p.d.f. of X and Y is given by

$$g(x, y) = \begin{cases} e^{-(x+y)}, & x \ge 0, y \ge 0 \\ 0, & \text{otherwise,} \end{cases}$$

find $P(X + Y \ge 1)$.

4. State the central limit theorem for independent and identically distributed random variables.

5. Define :

   (a) Wide sense stationary process

   (b) Markov process.

6. Let $X(t)$ be a Poisson process with rate λ. Find $E\{(X(t) - X(s))^2\}$ for $t > s$.

7. Define :

(a) Reliability function

(b) Hazard function.

8. Consider a system with five components connected in series. If each component has reliability 0.95, find the reliability of the system.

9. Define :

(a) Completely randomized design

(b) Mean square.

10. A plastic manufacturer extrudes blanks for use in the manufacture of eyeglass temples. Specification require that the thickness of these blanks have $\mu = 0.150$ inch and $\sigma = 0.002$ inch. Use the specifications to calculate a central line and three-sigma control limits for an $\overline{X}$ chart with $n = 5$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Three machines A, B and C produce identical items. Of their respective output 5%, 4%, and 3% of the items are faults. On a certain day A has produced 25%, B has produced 30% and C has produced 45% of the total output. An item selected at random is found to be faulty. What are chances that it was produced by C? (6)

(ii) If a random variable X has $E(X) = 2$ and $\text{Var}(X) = 4$, find (1) $E(X^2 + 3X + 2)$, (2) $\text{Var}(2X + 1)$. (6)

(iii) Find the probability density function of the random variable $Y = 3X + 1$ when X is a continuous random variable with p.d.f. (4)

$$f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Or

(b) (i) The p.d.f. for the random variable X is

$$f(x) = \begin{cases} 12.5x - 1.25, & 0.1 \le x \le 0.5 \\ 0, & \text{otherwise.} \end{cases}$$

Find (6)

(1) C.D.F. $F(X)$ for X

(2) $P(0.2 \le X \le 0.3)$

(3) $E(X)$.

(ii) Let X be the length in minutes of a long-distance telephone conversation. The p.d.f. for X is given by

$$f(x) = \begin{cases} \frac{1}{10} e^{-\frac{x}{10}}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

(1) Find the moment generating function $M_X(t)$ for X

(2) Use $M_X(t)$ to find the average length of such a call

(3) Find the variance and standard deviation for X. (6)

(iii) For the geometric distribution $P(X = k) = \frac{1}{2^k}, k = 1, 2, 3, \cdots$, prove that Chebyschev's inequality gives $P(|X - 2| > 2) < 0.5$, while the actual probability is $\frac{1}{16}$. (4)

12. (a) (i) Let X and Y be two discrete random variables with Joint probability mass function.

$$P(X = x, Y = y) = \begin{cases} \frac{x + 2y}{18}, & x = 1, 2, y = 1, 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find the correlation coefficient ρ_{XY} for the random variables X and Y. (8)

(ii) If the joint p.d.f. of the random variables X and Y is given by

$$f(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{otherwise,} \end{cases}$$

find the p.d.f. of the random variable $U = \frac{X}{Y}$. (8)

Or

(b) (i) If the joint p.d.f. of the random variables X and Y given by

$$f(x, y) = \begin{cases} 24xy, & 0 < x < 1, 0 < x + y < 1 \\ 0, & \text{otherwise,} \end{cases}$$

find $P(X \le x / Y = y)$. (6)

(ii) If X and Y are two random variables with the j.p.d.f.

$$f(x, y) = \begin{cases} 6\, e^{-(3x+2y)}, & x > 0, y > 0 \\ 0, & \text{otherwise,} \end{cases}$$

find the p.d.f. of $W = X + Y$. (10)

13. (a) (i) Prove that the difference of two independent Poisson processes is not a Poisson process. (4)

(ii) Let $\{X_n ; n \ge 1\}$ be a Markov chain with state space $S = \{1, 2, 3\}$ and one-step transition probability matrix

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/3 & 2/3 \end{bmatrix}.$$

(1) Is the chain is irreducible? Explain. (3)

(2) Is the chain periodic? If so, what is its period? (3)

(3) Is the state 2 recurrent? (3)

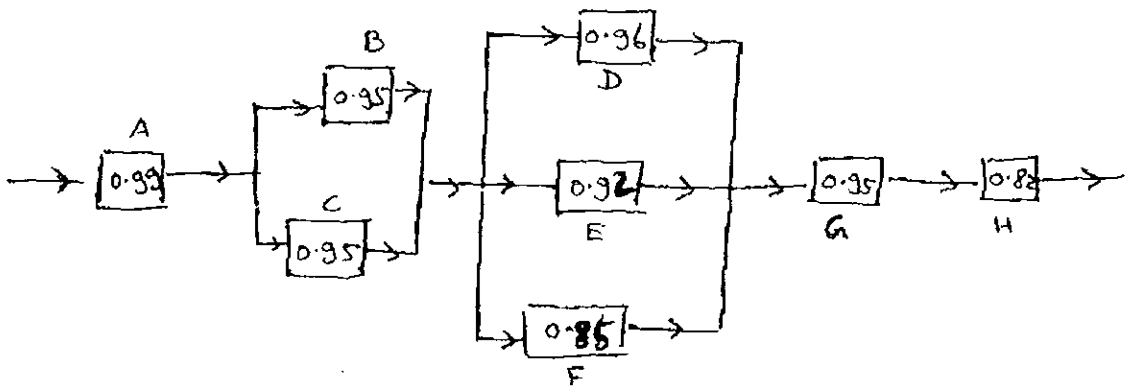
(4) Is the state 2 non-null? (3)

Or

(b) (i) For a random process $X(t) = Y \sin wt$, where Y is an uniform random variable in the interval -1 to 1 and w is a constant. Check whether the process is wide-sense stationery or not? (6)

(ii) Obtain the probability generating function of a birth and death process with λ and μ as birth and death rates, assuming the initial population size is one. Obtain its mean also. (10)

14. (a) (i) Consider a system consisting of eight independent components connected as shown in the figure :



Find the reliability of the system. (8)

(ii) Discuss the preventive maintenance for one-unit system and obtain its mean time to system failure (MTTSF). (8)

Or

(b) (i) The life length of a device is exponentially distributed. It is found that the reliability of the device for 100 hour period of operation is 0.90. How many hours of operation is necessary to get a reliability of 0.95? (6)

(ii) Discuss the reliability analysis for 2-unit parallel system with repair. (10)

15. (a) An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following "cleanness" readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines :

	Engine 1	Engine 2	Engine 3
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	48	50	55
Detergent D	42	37	49

Looking on the detergents as treatments and the engines as blocks, obtain the appropriate analysis of variance table and test at the 0.01 level of significance whether there are differences in the detergents or in the engines. (16)

Or

(b) A former wishes to test the effects of four difference fertilizers, A, B, C, D on the yield of wheat. In order to eliminate sources of error due to variability in soil fatality he uses the fertilizers in a Latin square arrangement as indicated in the following table, where the numbers indicate yields in bushels per unit area :

A 18	C 21	D 25	B 11
D 22	B 12	A 15	C 19
B 15	A 20	C 23	D 24
C 22	D 21	B 10	A 17

Perform an analysis of variance to determine if there is a significant difference between the fertilizers at $\alpha = 0.05$ levels of significance. (16)
