Reg. No. : [| | | | | | | | | | |]

Q 2378

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

First Semester

Civil Engineering

MA 131 — MATHEMATICS — I

(Common to all branches)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If A is an orthogonal matrix, prove that A^t and A^{-1} are orthogonal matrices.

2. State Cayley-Hamilton theorem.

3. Find the direction cosines of the line joining the points $(1,2,-3)$ and $(-4,0,7)$.

4. Find the centre and radius of the sphere $2x^2+2y^2+2z^2-6x+8y-8z-1=0$.

5. Find the curvature of the curve $2x^2+2y^2+5x-2y+1=0$.

6. Find the envelope of the family of straight lines $y = mx + \frac{a}{m}$ for different values of m.

7. Find $\frac{dy}{dx}$, if $x^3+y^3 = 3ax^2y$.

8. Write down functional determinant of u, v, w with respect to x, y and z.

9. Solve : $\frac{d^2x}{dt^2}+n^2x=0$.

10. Solve $(x^2D^2-3xD)y=0$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find A^4 if $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form given below to its canonical form by an orthogonal reduction $3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$. (16)

12. (a) (i) Prove that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect and find the coordinates of their point of intersection. Find also the equation of the plane containing them. (8)

(ii) Find the equation of the plane passing through the line of intersection of the planes $2x - y + 5z - 3 = 0$, $4x + 2y - z + 7 = 0$ and parallel to the z-axis. (8)

Or

(b) (i) Find the equation of the sphere that passes through the circle $x^2 + y^2 + z^2 + 3x + y + 2z - 2 = 0, x + 3y - 2z + 1 = 0$ cuts orthogonally the sphere $x^2 + y^2 + z^2 + x - 3z - 2 = 0$. (8)

(ii) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. (8)

13. (a) (i) Show that the radius of curvature of the curve $r = a(1 + \cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$. (8)

(ii) Find the envelope of the straight lines represented by the equation $x\cos\alpha + y\sin\alpha = a\sec\alpha$ where α is the parameter. (8)

Or

(b) (i) Show that the radius of curvature at the point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $3a\sin\theta\cos\theta$. (8)

(ii) Show that the equation of the evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$. (8)

14. (a) (i) If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y}$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$. (8)

(ii) Investigate the maxima and minima, if any, of the function $f(x,y) = xy + (x+y)(1-2x-3y)$. (8)

Or

(b) (i) Expand $e^x\log(1+y)$ in powers of x and y upto the third degree terms using Taylor's theorem. (8)

(ii) A rectangular box open at the top is to have a capacity of 108 cu.ms. Find the dimensions of the box requiring least material for its construction. (8)

15. (a) (i) Solve : $\dfrac{dx}{dt} + 2x - 3y = t$; $\dfrac{dy}{dt} - 3x + 2y = e^{2t}$. (8)

(ii) Solve $\dfrac{d^2y}{dx^2} + a^2y = \sec ax$, by using method of variation of parameters. (8)

Or

(b) (i) Solve $(D^2 - 4D + 3)y = \sin 3x\cos 2x$. (8)

(ii) Solve $x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + 5y = x\cos(\log x)$. (8)
