Reg. No. : | | | | | | | | | | | |

Q 2379

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Second Semester

Civil Engineering

MA 132 — MATHEMATICS — II

(Common to all branches except Information Technology)

Time : Three hours Maximum : 100 marks

Use of statistical tables permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Sketch roughly the region of integration for the double integral $\int_0^1 \int_0^x f(x, y)dydx$.

2. Prove that $\beta(m+1, n) + \beta(m, n+1) = \beta(m, n)$.

3. Define Laplacian operator and harmonic function.

4. State Gauss divergence theorem.

5. State the orthogonal property of an analytic function.

6. Find the critical points for the transformation $w^2 = (z-\alpha)(z-\beta)$.

7. Find the principal part and residue at the pole of the function $f(z) = \frac{2z+3}{(z+2)^2}$.

8. Evaluate $\int_c \frac{dz}{z+4}$ where c is the circle $|z| = 2$.

9. Write down the formula for μ_3 interms of moment about any point.

10. What are the uses of χ^2 distribution?

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ and hence evaluate it. (8)

(ii) Express the volume of the sphere $x^2 + y^2 + z^2 = a^2$ as a volume integral and hence evaluate it. (8)

Or

(b) (i) Evaluate $\iint r^3\,dr\,d\theta$ over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$. (8)

(ii) Show that $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx = \frac{\lfloor n+1}{(m+1)^{n+1}}$. (8)

12. (a) (i) Given $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Then prove that $r^n \vec{r}$ is solenoidal only when $n = -3$, but irrotational for all values of n. (8)

(ii) Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\,\hat{j} - y^2 z\hat{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (8)

Or

(b) (i) Find a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at $(1, -1, 2)$. (8)

(ii) Verify Gauss divergence theorem for $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ where S is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0$ and $z = c$. (8)

13. (a) (i) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

(ii) Show that the transformation $w = \frac{1}{z}$ transforms all circles and straight lines in the z-plane into circles or straight lines in the w-plane. (8)

Or

(b) (i) Show that under the mapping $w = \frac{i-z}{i+z}$, the image of the circle $x^2 + y^2 < 1$ is the entire half of the w – plane to the right of the imaginary axis. (8)

(ii) Show that $v = e^x (x\cos y - y\sin y)$ is a harmonic function. Find the corresponding analytic function $f(z)$. (8)

14. (a) (i) Expand $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ in a Laurent's series for (1) $2 < |z| < 3$ and (2) $|z| > 3$. (8)

(ii) By the method of contour integration prove that

$$\int_0^{2\pi} \frac{d\theta}{1 - 2a\cos\theta + a^2} = \frac{2\pi}{1 - a^2}, \quad \text{if } 0 < a < 1. \tag{8}$$

Or

(b) (i) Using Cauchy's integral formula, evaluate $\int_c \frac{z+4}{z^2 + 2z + 5} dz$ where c is the circle $|z + 1 - i| = 2$. (8)

(ii) Prove that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)} = \frac{\pi}{a+b}$, $a, b > 0$ by using contour integration. (8)

15. (a) (i) Given the density function of a continuous R.V. is

$f(x) = \lambda e^{-\lambda x}$, $x \geq 0$, $\lambda > 0$

$= 0$, otherwise

Find the third and fourth order central moments. (8)

(ii) A sample of 900 members has a mean 3.4 and S.D 2.61. Is the sample from a large population of mean 3.25 and S.D 2.61? Assuming population as normal, find the 95% confidence limits for its mean. (8)

Or

(b) (i) Find the coefficient of correlation and obtain the lines of regression from the data given below : (8)

x :	62	64	65	69	70	71	72	74
y :	126	125	139	145	165	152	180	208

(ii) Two researchers A and B adopted different techniques while rating the students level. Can you say that the techniques adopted by them are significant? (8)

Researchers	Below average	Average	Above average	Genius	Total
A	40	33	25	2	100
B	86	60	44	10	200
Total	126	93	69	12	300