Reg. No. : | | | | | | | | | | | |

Q 2381

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Third Semester

Civil Engineering

MA 231 — MATHEMATICS — III

(Common to All Branches except Bio-Medical Engineering/Civil Engineering and Computer Based Construction/Fashion Technology/Industrial Biotechnology/ Textile Chemistry)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the general solution of $\frac{\partial^2 z}{\partial x^2} = 0$.

2. Form the partial differential equation by eliminating a and b from $z = a(x+y)+b$.

3. If $\cos^3 t = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt)$ in $0 \le t \le 2\pi$, find the sum of the series $\frac{a_0^2}{4} + \sum_{1}^{\infty}\left(a_n^2 + b_n^2\right)$.

4. The Fourier series of x^2 in (0, 2) and that of $(x+2)^2$ in $(-2, 0)$ are identical or not. Give reasons.

5. State any two assumptions involved in deriving one dimensional wave equation.

6. How many conditions are required to solve

   $$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

7. Give an example of a function which has Laplace transform but it is not continuous.

8. If $L[f(t)] = \frac{1}{(s-2)^2}$ then find $\underset{t \to 0}{Lt} f(t)$.

9. Find the fourier sine transform of $e^{-ax}, a > 0$.

10. State the shifting properties on fourier transform.

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) Form the p.d.e of the family of planes that are at constant distance k from the origin.

(ii) Solve $p^2 + q^2 = z^2(x^2 + y^2)$.

Or

(b) (i) Solve $(y+z)p + (z+x)q = x + y$.

(ii) Solve $(D^2 + DD' - 6D'^2)z = y\cos x$.

12. (a) (i) Find the fourier series expansion of $f(x) = x^2$ in $(0, 2l)$.

(ii) Find the half range sine series expansion of $f(x) = \frac{\pi}{2} - x$ in $(0, \pi)$ and deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Or

(b) (i) Find the half range cosine series expansion of $f(x) = x$ in $(0, \pi)$.

(ii) Find upto the first two harmonics in the fourier series of $y = f(x)$ in $(0, 360)$ given in the following tabular value

x	0°	60°	120°	180°	240°	300°	360°
y	2	2.1	3	3.2	2.5	2.2	2

13. (a) (i) Find the Laplace transform of $t\,e^{2t}\sin 3t$ and $\frac{\sin 3t}{t}$.

(ii) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases} \quad \text{and } f(t + 2a) = f(t)$$

Or

(b) (i) Find the inverse Laplace transform of $\log\left[\frac{s^2+1}{s(s+1)}\right]$.

(ii) Using the Laplace transform solve the differential equation

$$y'' + 3y' + 2y = e^{-t}, \quad y(0) = 1, \; y'(0) = 0$$

14. (a) A string of length $2l$, fastened at both ends. Motion is started by displacing the string into the form $y = kx(2l - x)$ and then releasing it from this position at time $t = 0$. Find the displacement of the point of the string at a distance x from one end at time 't'.

Or

(b) A rectangular plate of sides a and b has its faces insulated and the edges $y = 0$ and $y = b$ and $x = 0$ are kept at 0°C and the edge $x = a$ is kept at temperature $k(2y - b)$. Find the steady state temperature distribution in the plate.

15. (a) (i) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

and hence prove that $\int_0^\infty \frac{\sin^4 x}{x^4} dx$.

(ii) Find the Fourier cosine transform of

$$f(x) = e^{-a^2 x^2}$$

Or

(b) (i) Find the Fourier sine transform of $f(x) = e^{-ax}$ and hence evaluate $\int_0^\infty \frac{x^2\, dx}{(a^2 + x^2)^2}$.

(ii) State and prove convolution theorem on Fourier transform.