

8. Mention the multistep methods available for solving ordinary differential equation.
9. Write down the finite difference scheme for solving the Poisson's equation.
10. Obtain the finite difference scheme for the differential equation $2 \frac{d^2 y}{dx^2} + y = 5$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the real root of the equation $X \log_{10} X - 1.2 = 0$ correct to four places of decimal using false position method. (8)
- (ii) Using Gauss-Jordan method, solve the following system of equations

$$\begin{aligned} 2x - y + 3z &= 8 \\ -x + 2y + z &= 4 \\ 3x + y - 4z &= 0. \end{aligned} \quad (8)$$

Or

- (b) (i) Solve by Gauss-Jacobi method, the following equations
- $$\begin{aligned} 4x_1 + x_2 + x_3 &= 6 \\ x_1 + 4x_2 + x_3 &= 6 \\ x_1 + x_2 + 4x_3 &= 6. \end{aligned} \quad (8)$$

- (ii) Find the largest eigen values and eigen vector of the matrix by

power method $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$. (8)

12. (a) Use Lagrange formula to calculate $f(3)$ from the following table.

$$\begin{array}{l} x: \quad 0 \quad 1 \quad 2 \quad 4 \quad 5 \quad 6 \\ f(x): \quad 1 \quad 14 \quad 15 \quad 5 \quad 6 \quad 19 \end{array} \quad (16)$$

Or

- (b) (i) Using Newton's divided difference formula, find the value of $f(8)$ and from the following table :

x :	4	5	7	10	11	13	
$f(x)$:	48	100	294	900	1210	2028	(8)

- (ii) Given

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

Estimate $f(7.5)$. Use Newton's formula. (8)

13. (a) (i) Find the value of $f'(8)$ from the table given below : (8)

x	6	7	9	12
$f(x)$	1.556	1.690	1.908	2.158

- (ii) Use Simpson's 1/3 rule to estimate the value of $\int_1^5 f(x) dx$ given (8)

x	1	2	3	4	5
$f(x)$	13	50	70	80	100

Or

- (b) (i) Find the first and second derivatives of the function $f(x) = x^3 - 9x - 14$ at $x = 3.0$ using the values given below : (8)

x	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$	-14	-10.03	-5.296	-0.256	-6.672	14

- (ii) Find the value of the following integral using Gaussian quadrature technique $\int_3^5 \frac{4}{(2x^2)} dx$. (8)

14. (a) (i) Solve numerically $\frac{dy}{dx} = x + y$ when $y(1) = 0$ using Taylor's series upto $x = 1.2$ with $h = 0.1$. (8)

- (ii) Using fourth order Runge-Kutta method, solve the following equation taking each step of $h = 0.1$. Given $y(0) = 3$, $\frac{dy}{dt} = \left[\frac{4t}{y} - t \cdot y \right]$. Calculate y for $x = 0.1$ and 0.2 (8)

Or

(b) (i) Apply the modified Euler's method to find $y(0.2)$ and $y(0.4)$, given that $y' = x^2 + y^2$, $y(0) = 1$. Take $h = 0.2$. (8)

(ii) Using Milne's method, find $y(4.4)$ given $5xy' + y^2 - 2 = 0$. Given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$. (8)

15. (a) Find the solution of the initial boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad 0 \leq x \leq 1$$

Subject to the initial conditions

$$u(x, 0) = \sin \pi x, \quad 0 \leq x \leq 1$$

$$(\partial u / \partial t)(x, 0) = 0, \quad 0 \leq x \leq 1$$

and the boundary conditions

$$u(0, t) = 0, u(1, t) = 0, \quad t > 0$$

by using (i) the explicit scheme (ii) the implicit scheme. (16)

Or

(b) Solve

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

given $u(0, t) = 0$, $u(4, t) = 0$, $u(x, 0) = x(4 - x)$ assuming $h = k = 1$.

Find the values of u upto $t = 1.5$. (16)