Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐

R 3425

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Fourth Semester

Electronics and Communication Engineering

MA 1254 — RANDOM PROCESSES

(Regulation 2004)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. In a coin tossing experiment, if the coin shows head, 1 dice is thrown and the result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. What is the probability that the recorded number will be 2?

2. Find the moment generating function of binomial distribution.

3. Find the mean and variance of Geometric distribution.

4. If X is a Gaussian random variable with mean zero and variance σ^2, find the probability density function of $Y = |X|$.

5. The joint probability density function of the random variable (X,Y) is given by

   $$f(x, y) = K\,xy\,e^{-(x^2+y^2)}, \quad x > 0, \ y > 0$$

   Find the value of K.

6. If the probability density function of X is $f_X(x) = 2x$, $0 < x < 1$, find the probability density function of $Y = 3X + 1$.

7. Prove that sum of two independent Poisson process is also Poisson.

8. Define wide-sense stationary process.

9. Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}$.

10. If the power spectral density of a WSS process is given by

$$S(w) = \begin{cases} \dfrac{b}{a}(a - |w|), & |w| \le a \\ 0, & |w| > a \end{cases}$$

Find the autocorrelation function of the process.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has the following probability distribution.

x :	-2	-1	0	1	2	3
$p(x)$:	0.1	k	0.2	$2k$	0.3	$3k$

Find

(1) The value of k,

(2) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$

(3) Find the cumulative distribution of X and

(4) Evaluate the mean of X.

(ii) An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball?

Or

(b) (i) If the density function of a continuous random variable X is given by

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0, & elsewhere \end{cases}$$

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Find

(1) the value of a

(2) the cumulative distribution function of X

(3) If x_1, x_2 and x_3 are 3 independent observations of X. What is the probability that exactly one of these 3 is greater than 1.5?

(ii) Find the moment generating function of Poisson distribution and hence find mean and variance of the same.

12. (a) (i) Find first four central moments of normal distribution.

(ii) VLSI chips, essential to the running of a computer system, fail in accordance with a poisson distribution with the rate of one chip in about 5 weeks. If there are two spare chips on hand, and if a new supply will arrive in 8 weeks. What is the probability that during the next 8 weeks the system will be down for a week or more, owing to a lack of chips?

Or

(b) (i) The lifetime X in hours of a component is modeled by a weibull distribution with shape parameter $\alpha = 2$. Starting with a large number of components, it is observed that 15% of the components that have lasted 90 hours fail before 100 hours. Determine the scale parameter λ.

(ii) Find mean and variance of Gamma distribution and hence find mean and variance of exponential distribution.

13. (a) (i) The random variables X and Y are statistically independent having a gamma distribution with parameters $\left(m, \dfrac{1}{2}\right)$ and $\left(n, \dfrac{1}{2}\right)$, respectively. Derive the probability density function of a random variable $U = \dfrac{X}{(X + Y)}$.

(ii) State and prove central limit theorem.

Or

(b) (i) Find the correlation coefficient for the following data :

X	10	14	18	22	26	30
Y	18	12	24	6	30	36

(ii) The joint probability mass function of (X, Y) is given by $p(x, y) = K(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find all the marginal and conditional probability distributions.

14. (a) (i) Define random process. Classify it with an example.

(ii) The process $\{X(t)\}$ whose probability distribution under certain conditions is given by

$$P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1 + at)^{n+1}}, & n = 1, 2, \ldots \\ \dfrac{at}{1 + at}, & n = 0 \end{cases}$$

Show that it is not stationary.

Or

(b) (i) If $X(t) = Y \cos wt + Z \sin wt$, where Y and Z are two independent normal random variables with $E(Y) = E(Z) = 0$, $E(Y^2) = E(Z^2) = \sigma^2$ and w is a constant, prove that $\{X(t)\}$ is a strict sense stationary process of order 2.

(ii) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

15. (a) (i) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2\gamma|\tau|}$. Determine the power density spectrum of the random telegraph signal.

(ii) Show that in an input-out system the energy of a signal is equal to the energy of its spectrum.

Or

(b) (i) State and prove Wiener-Khinchine theorem.

(ii) A random process $\{X(t)\}$ is given by $X(t) = A \cos pt + B \sin pt$, where A and B are independent RVS such that $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = \sigma^2$. Find the power spectral density of the process.

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