D 4522

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Annual Pattern — First Year

(Regulation 2004)

Civil Engineering

MA 1 X 01 — ENGINEERING MATHEMATICS – I

(Common to all branches)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

State Cayley – Hamilton theorem.

Find the radius of curvature at $x = \dfrac{\pi}{2}$ on the curve $y = 4\sin x - \sin 2x$.

Solve : $\dfrac{d^2 y}{dx^2} - 6\dfrac{dy}{dx} + 13y = 0$.

Write down the differential equation expressing Newton's law of cooling.

Evaluate $\displaystyle\int_{1}^{2}\int_{0}^{x} \dfrac{1}{x^2 + y^2}\, dx\, dy$.

Find $curl\ \vec{F}$ if $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$.

Find where the following function ceases to be analytic.

$$f(z) = \dfrac{z^2 - 4}{z^2 + 1}.$$

8. Evaluate $\oint_C \dfrac{z+2}{z} dz$ where C is the circle $|z| = 2$ in the z–plane.

9. Find the Laplace transform of $f(t) = \cos^2 3t$.

10. Find $f(t)$ if $L(f(t)) = \dfrac{s}{(s+2)^2}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and the eigen vectors of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

   (ii) Reduce the quadratic form below to its normal form by an orthogonal reduction. $q = 3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1 x_2 - 2x_2 x_3$.

Or

(b) (i) Find the radius of curvature at the point $(a, 0)$ on the curve $xy^2 = a^3 - x^3$.

   (ii) Find the minimum values of $x^2 y z^3$ subject to the condition $2x + y + 3z = a$.

12. (a) (i) Solve : $(D^2 - 4D + 3)y = \sin 3x \cos 2x$.

   (ii) Solve : $\dfrac{dx}{dt} + 2x - 3y = t$, $\dfrac{dy}{dt} - 3x + 2y = e^{2t}$.

Or

(b) (i) An object whose temperature is 75°C cools in an atmosphere of constant temperature 25°C at the rate of $K\theta$, θ being the excess temperature of the body over that of the atmosphere. If after 10 minutes, the temperature of the object falls to 65°C, find its temperature after 20 minutes. Also find the time required to cool down to 55°C.

   (ii) A cantilever beam of length l, with uniform load w per unit length has a concentrated load w at the free end. Taking the origin at the fixed end, find the equation of the elastic curve of the beam. Determine the maximum deflection and the value of the clamping couple at the fixed end.

13. (a) (i) Change the order of integration in $\int\limits_0^1 \int\limits_x^{\sqrt{2-x^2}} \dfrac{x}{\sqrt{x^2+y^2}}\,dx\,dy$ and hence evaluate it.

(ii) Find the volume of that portion of the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$ which lies in the first octant using triple integration.

Or

(b) Verify Stoke's theorem for $\vec{F}=x^2\vec{i}+xy\,\vec{j}$ in the square region in the XOY plane bounded by the lines $x=0$, $y=0$, $x=a$, $y=a$.

14. (a) (i) Show that an analytic function with
   (1) constant real part is a constant; and
   (2) constant modulus is a constant.

(ii) Find the bilinear transformation that maps the points $1+i$, $-i$, $2-i$ of the z-plane into the points $0,1,i$ of the w-plane.

Or

(b) (i) Use contour integration to prove that $\int\limits_0^\infty \dfrac{\sin mx}{x}\,dx=\dfrac{\pi}{2}$, when $m>0$.

(ii) Find the residues of $f(z)=\dfrac{z^2}{(z-1)(z+2)^2}$ at its isolated singularities using Laurent's series expansion.

15. (a) (i) Verify the initial and final value theorems for the function $f(t)=1+e^{-t}(\sin t+\cos t)$.

(ii) Find the Laplace transform of the 'meoander function' defined as $f(t)=1$ when $0<t<\dfrac{1}{2}a$ and $f(t)=-1$ when $\dfrac{1}{2}a<t<a$ and $f(t)$ is periodic with period 'a' so that $f(t+a)=f(t)$ for $t>0$.

Or

(b) (i) Solve the equations $\dfrac{dx}{dt}-y=e^t$, $\dfrac{dy}{dt}+x=\sin t$ given that $x(0)=1$ and $y(0)=0$ using Laplace transforms.

(ii) Apply the convolution theorem to find $L^{-1}\left\{\dfrac{1}{s(s^2-a^2)}\right\}$.

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