A 1350

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Sixth Semester

Electronics and Instrumentation Engineering

MA 038 — NUMERICAL METHODS

(Common to Chemical Engineering / Leather Technology / Textile Technology)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Name the two different methods to find the solution of simultaneous linear algebraic equations.

2. Write all possible types of initial vectors to determine the largest eigen value and the corresponding eigen vector of a matrix of size 2 × 2.

3. Show that the divided differences for the given points (x_0, y_0), (x_1, y_1) and (x_2, y_2) are symmetrical in their arguments.

4. Find $y(1)$, given

x :	0	3	4
y :	12	6	8

5. Write the formula for the derivative $\frac{dy}{dx}$ at the point x, using Bessel's interpolation formula.

6. Evaluate : $\int_0^1 \frac{dx}{1+x}$, using trapezoidal rule, by taking $\Delta x = 0.25$.

7. Use Euler's modified formula to find the solution of y at $x = 0.1$, in solving $\frac{dy}{dx} = 1 + xy$, with $y(0) = 2$ and starting value of $y(0.1) = 2.1$ (only two iterations).

8. State Milne's predictor and corrector formulae.

9. State the diagonal five point formula in solving $\nabla^2 u = 0$.

10. State the mesh ratio parameter in the solution of $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{dx^2}$ and also its range of validity.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Apply the method of iteration to solve the equation $3x - \log_{10} x = 6$ in order to determine the root, correct to four places of decimals. (8)

(ii) Solve the following system of equations using Jacobi : iteration method, correct to three places of decimals :

$3x + 4y + 15z = 54.8,\ x + 12y + 3z = 39.66,\ 10x + y - 2z = 7.74$ (8)

Or

(b) (i) Evaluate $\sqrt{12}$ to four places of decimals, by Newton Raphson method. (6)

(ii) Using Jacobi's method, find all the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$. (10)

12. (a) (i) A function $y = f(x)$ is given by the following table. Find $f(0.2)$ and $f(5.4)$ by suitable formulae : (8)

x :	0	1	2	3	4	5	6
$f(x)$:	176	185	194	203	212	220	229

(ii) Find the equation of the cubic curve which passes through the points (4, −43), (7, 83), (9, 327) and (12, 1053). Hence find $f(10)$, using divided difference interpolation formula. (8)

Or

(b) (i) Using Stirlings formula to compute $y(35)$ and suitable formula to compute $y(42)$, given that $y\ (10) = 600$, $y(20) = 512$, $y(30) = 439$, $y(40) = 346$ and $y(50) = 243$. (8)

(ii) The values $f(x)$ are known at $x = a$, $x = b$ and $x = c$ as $f(a)$, $f(b)$ and $f(c)$ respectively. Use Lagranges interpolation formula to express $f(x)$ interms of $f(a)$, $f(b)$ and $f(c)$. Also show that maximum or minimum Lagranges interpolation polynomial is attained at $x = \frac{\Sigma\left(b^2 - c^3\right) f(a)}{2\Sigma (b-c)\ f(a)}$. (8)

13. (a) (i) Find the gradient of the road at x = 450, 1700 and 900 from the data given below : (10)

x :	0	300	600	900	1200	1500	1800
$y(x)$:	135	149	157	183	201	205	193

(ii) Apply Simpson's rule to evaluate the integral $I = \int_{y=2}^{2.6} \int_{x=4}^{4.4} \frac{dx\, dy}{xy}$, with $\Delta x = 0.2,\ \Delta y = 0.3$. (6)

Or

(b) (i) Given that

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$y(x)$:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ at $x = 1.1$ and 1.6. (8)

(ii) Evaluate : $\int_0^1 \frac{dx}{1+x}$ using

(1) Trapezoidal rule

(2) Simpson's 1/3 and 3/8 rules.

Find the absolute error in each method by comparing the actual integration value. [by taking six equal parts]. (8)

14. (a) (i) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y(0)=1$, find y for $x=0.1$ with step size $\Delta x = 0.02$ by Euler's method. (6)

(ii) Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1)=1$, $y(1.1)=1.233$, $y(1.2)=1.548$, $y(1.3)=1.979$, evaluate $y(1.4)$ by Adams-Bashforth method. (10)

Or

(b) (i) Given $\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$, $y(0)=1$, $y'(0)=0$, find the value of y at $x=0.1$ by applying Runge-Kutta method of fourth order. (10)

(ii) Use Milne's method find $y(4.4)$ given $5x\frac{dy}{dx} + y^2 - 2 = 0$, given $y(4)=1$, $y(4.1)=1.0049$, $y(4.2)=1.0097$ and $y(4.3)=1.0143$. (6)

15. (a) Solve : $\nabla^2 u = -10\left(x^2+y^2+10\right)$, where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ over the square with sides $x=0$, $x=3$, $y=0$ and $y=3$; with $u=0$ on the boundaries and mesh length 1 on both directions x and y. [Use Gauss-Seidel iteration technique]. (16)

Or

(b) Solve by Crank-Nicholson's method, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, $t > 0$ given that $u(0, t) = 0$, $u(1, t) = 0$ and $u(x, 0) = 100x(1-x)$. Compute for one step with $\Delta x = 0.25$. (16)
