

B 2323

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Second Semester

Aeronautical Engineering

(Common to all branches except Information Technology)

MA 132 — MATHEMATICS — II

Time : Three hours

Maximum : 100 marks

Use of Statistical tables permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$.
2. Evaluate $\int_0^{\infty} x e^{-x^2} dx$.
3. If $\phi = x^2 + y^2 + z^2 - 8$, then find grad ϕ at $(2, 0, 2)$.
4. State Gauss divergence theorem.
5. Write down the formula for finding an analytic function $f(z) = u + iv$, whenever the real part is given, by using Milne Thomson Method.
6. Define Bilinear transformation.
7. State Cauchy's integral formula.
8. Determine the residues at poles of the function $f(z) = \frac{z+4}{(z-1)(z-2)}$.
9. Define Type I and Type II errors.
10. Write down the χ^2 value, for a 2×2 contingency table $\begin{array}{c|c} a & b \\ \hline c & d \end{array}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find, by double integration, the area between the parabola $y^2 = 4ax$ and the line $y = x$. (8)

- (ii) Change the order of integration and evaluate the integral.

$$\int_0^a \int_x^a (x^2 + y^2) dy dx \quad (8)$$

Or

- (b) (i) Find, by double integration, the area of the cardioid $r = a(1 + \cos \theta)$ (8)

- (ii) Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma m + n}$. (8)

12. (a) (i) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\text{div grad}(r^n) = n(n+1)r^{n-2}$.
Hence deduce that $\text{div grad}\left(\frac{1}{r}\right) = 0$. (8)

- (ii) Verify stoke's theorem for $\vec{F} = -y\hat{i} + 2yz\hat{j} + y^2\hat{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$. (8)

Or

- (b) (i) Prove that $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \text{Curl } \vec{A} - \vec{A} \cdot \text{Curl } \vec{B}$. Hence deduce that $\vec{A} \times \vec{B}$ is solenoidal where \vec{A} and \vec{B} are irrotational. (8)

- (ii) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative field. Find the scalar potential. (8)

13. (a) (i) Show that the function $v = e^{-x}(x \cos y + y \sin y)$ is harmonic and find the corresponding analytic function $f(z) = u + iv$. (8)

- (ii) Discuss the conformal mapping $w = z^2$. (8)

Or

(b) (i) If $f(z) = u + iv$ is an analytic function of z , then prove that $\nabla^2 [\log |f'(z)|] = 0$. (8)

(ii) Show that under the mapping $w = \frac{i-z}{i+z}$ the image of the circle $x^2 + y^2 < 1$ is the entire half of the w -plane to the right of the imaginary axis. (8)

14. (a) (i) Obtain the Laurent's series expansion for the function $f(z) = \frac{4z}{(z^2 - 1)(z - 4)}$ in the regions $2 < |z - 1| < 3$ and $|z - 1| > 4$. (8)

(ii) Using the method of contour integration, prove that $\int_0^{2\pi} \frac{d\theta}{5 - 4 \cos \theta} = \frac{2\pi}{3}$. (8)

Or

(b) (i) Evaluate $\int_{|z|=3} \frac{\cos \pi z^2 + \sin \pi z^2}{(z+1)(z+2)} dz$, using Cauchy's residue theorem. (8)

(ii) Evaluate $\int_0^{\infty} \frac{x \sin x}{x^2 + 1} dx$ by contour integration. (8)

15. (a) (i) Find the coefficient of correlation and obtain the lines of regression from the data given below : (8)

| | | | | | | | | |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|
| $x :$ | 62 | 64 | 65 | 69 | 70 | 71 | 72 | 74 |
| $y :$ | 126 | 125 | 139 | 145 | 165 | 152 | 180 | 208 |

(ii) The average income of 100 persons was Rs. 210 with a S.D. of Rs. 10 in one region of a city. For another sample of 150 persons, from a different region the average income was Rs. 220 with S.D. of Rs. 12. The S.D. of incomes of the people of the city was Rs. 11. Test whether there is any significant difference between the average incomes of the two regions. (8)

Or

- (b) (i) A sample of 14 tyres had a mean life 60,000 kilometers. Test whether it meets the population mean 59,500 kms at 5% level.
- (ii) Fit a binomial distribution for the given data and test goodness of fit.

| | | | | | |
|------|---|----|----|----|---|
| $x:$ | 0 | 1 | 2 | 3 | 4 |
| $y:$ | 4 | 44 | 66 | 38 | 8 |
