B 2324

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Third Semester

Civil Engineering

MA 231 : MATHEMATICS — III

(Common to all branches except Bio-Medical Engineering/Civil Engineering and Computer based construction/Fashion Technology/Industrial Biotechnology/ Textile Chemistry)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Obtain partial differential equation by eliminating the arbitrary functions f and g from $z = f(x+it)+g(x-it)$.

2. Solve $4\frac{\partial^2 z}{\partial x^2} - 12\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = 0$.

3. State Dirichlet's conditions.

4. If $f(x)$ is discontinuous at $x = a$ what value does its Fourier series represent at that point.

5. Write the initial conditions of the wave equation if the string has an initial displacement but no initial velocity.

6. Write the partial differential equation governing one dimensional heat conduction.

7. Find the inverse Laplace transform of $\cot^{-1} s$.

8. State the final value theorem on Laplace transform.

9. If $F[f(x)] = \bar{f}(s)$, prove that $F[f(x-a)] = e^{-ias}\bar{f}(s)$.

10. State convolution theorem on Fourier transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $(x + pz)^2 + (y + qz)^2 = 1$ (8)

(ii) Solve : $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = y \sin x$ (8)

Or

(b) Solve :

(i) $x\left(y^2 + z^2\right)p + y\left(z^2 + x^2\right)q = z\left(y^2 - x^2\right)$ (8)

(ii) $\left(4D^2 - 4DD' + D'^2\right)z = e^{3x-2y} + \sin x$ (8)

12. (a) (i) Obtain Fourier series of period $2l$ for $f(x)$ where (8)

$$f(x) = \begin{cases} l - x \text{ in } 0 < x \le l \\ 0 \quad \text{in } l \le x \le 2l \end{cases}$$

hence find the sum of $1 - \frac{1}{3} + \frac{1}{5} - + \cdots$ to ∞. (2)

(ii) Find the half range Fourier Sine series of $f(x) = x^2$ in $(0, \pi)$. (6)

Or

(b) Expand $f(x) = \begin{cases} \sin x & \text{in } 0 \le x \le \pi \\ 0 & \text{in } \pi \le x \le 2\pi \end{cases}$

as a Fourier series of periodicity 2π and hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$ to ∞. (16)

13. (a) A uniform string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string into the form of the curve $y = kx(l - x)$ and then releasing it from this position at time $t = 0$. Find the displacement of the point of the string at a distance x from one end at time t. (16)

Or

(b) A rectangular plate with insulated surfaces is 'a' cm wide and so long compared to its width that it may be considered infinite in length, without introducing an appreciable error. If the two long edges $x = 0$ and $x = a$ and the short edge at infinity are kept at temperature 0° C, while the other short edge $y = 0$ is kept at temperature $u_0 \sin^3 \frac{\pi x}{a}$, find the steady state temperature at any point (x, y) of the plate. (16)

14. (a) (i) Find the Laplace transforms of

(1) $\frac{e^{at} - \cos bt}{t}$

(2) $\sin t \, u_\pi(t)$, where $u_\pi(t)$ is unit step function. (3 + 3)

(ii) Solve the simultaneous equations

$D^2x - Dy = \cos t$ and

$Dx + D^2y = -\sin t$ where $D(\,) = \frac{d}{dt}(\,)$

given that $x = 1$, $Dx = 0$, $y = 0$, and $Dy = 1$ at $t = 0$. (10)

Or

(b) (i) Use convolution to find the inverse Laplace transform of

$$\frac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}$$ (6)

(ii) Find the Laplace transform of the square wave function

$f(t) = \begin{cases} k & \text{in } 0 \le t \le a \\ -k & \text{in } a \le t \le 2a \end{cases}$ and $f(t + 2a) = f(t)$ for all t. (10)

15. (a) (i) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| \ge 1 \end{cases}$

Hence evaluate $\int_0^\infty \left[\frac{\sin x - x\cos x}{x^3}\right] \cos\left(\frac{x}{2}\right) dx$ (10)

(ii) Find the Fourier sine transform of $f(x)$ defined as

$f(x) = \begin{cases} \sin x & \text{where } 0 < x < a \\ 0 & \text{where } x > a \end{cases}$ (6)

Or

(b) (i) Find the Fourier transform of $f(x)$ if

$$f(x) = \begin{cases} 1-|x| & \text{for } |x|<1 \\ 0 & \text{for } |x|>1 \end{cases}$$

Hence prove that $\int_0^\infty \frac{\sin^4 x}{x^4} = \frac{\pi}{3}$. (10)

(ii) If $F[f(x)] = \bar{f}(s)$ prove that $F[f(ax)] = \frac{1}{|a|}\bar{f}\left(\frac{s}{a}\right)$. (6)
