C 3274

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Second Semester

(Regulation 2004)

Civil Engineering

MA 1151 — MATHEMATICS — II

(Common to all branches)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^2 \int_0^y \frac{dx\,dy}{x^2+y^2}$.

2. Find $\iint dx\,dy$ over the region bounded by $x \geq 0,\ y \geq 0,\ x+y \leq 1$.

3. Prove that the vector $\vec{F} = (3x+2y+4z)\vec{i} + (2x+5y+4z)\vec{j} + (4x+4y-8z)\vec{k}$ is both solenoidal and irrotational.

4. Using Green's theorem prove that the area enclosed by a simple closed curve C is $\frac{1}{2}\int (x\,dy - y\,dx)$.

5. Verify the function $\phi(x, y) = e^x \sin y$ is harmonic or not.

6. Under the transformation $w = iz + i$ show that the half plane $x > 0$ maps into the half plane $w > 1$.

7. Evaluate $\int_C \frac{1}{2z-3}\,dz$ where C is $|z| = 1$.

8. Find the poles of $f(z) = \frac{1}{\sin \frac{1}{z-a}}$.

9. Find the Laplace transform of $e^{-2t}(1+t)^2$.

10. Find the inverse Laplace transform of $\frac{s+2}{s^2+2s+2}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dx\,dy$ and evaluate.

(ii) Find by triple integral the volume of the tetrahedron bounded by the planes $x=0,\ y=0,\ z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

Or

(b) (i) By transforming into cylindrical coordinates, evaluate the integral $\iiint (x^2+y^2+z^2)\,dx\,dy\,dz$, taken over the region of space defined by $x^2+y^2 \leq 1$ and $0 \leq z < 1$.

(ii) Find the area of the region D bounded by the parabolas $y = x^2$ and $x = y^2$.

12. (a) (i) Evaluate $\int_C \vec{f}\cdot d\vec{r}$ where $\vec{f} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ along the straight line joining (1, –2, 1) and (3, 2, 4).

(ii) Evaluate $\int_C (e^x\,dx + 2y\,dy - dz)$ by using Stoke's theorem, where C is the curve $x^2 + y^2 = 4,\ z = 2$.

Or

(b) (i) Evaluate $\iint_S \vec{f}\cdot\vec{n}\,dS$ where $\vec{f} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the $2x + y + 2z = 6$ in the first octant.

(ii) Use Gauss divergence theorem to evaluate $\iint_S \vec{f}\cdot\vec{n}\,dS$ where $\vec{f} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the surface bounding the region $x^2+y^2=4,\ z=0$ and $z=3$.

13. (a) (i) Find the real part of the analytic function whose imaginary part is $e^{-x}\left[2xy\cos y+(y^2-x^2)\sin y\right]$. Construct the analytic function.

(ii) Find the bilinear transformation which maps the points 1, i, -1 onto the points $0, 1, \infty$. Show that the transformation maps the interior of the unit circle of the z–plane onto the upper half of the w–plane.

Or

(b) (i) If $f(z)$ is analytic function prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4\,|f'(z)|^2$.

(ii) Find the image of $|z+2i|=2$ under the transformation $w=\frac{1}{z}$.

14. (a) (i) Using Cauchy's integral formula evaluate $\int_C \frac{e^z dz}{(z+2)(z+1)^2}$ where C is $|z|=3$.

(ii) Evaluate $\int_0^\infty \frac{\cos x}{1+x^2}\,dx$ using contour integration.

Or

(b) (i) Expand $\frac{1}{z(z-1)}$ as Laurent's series (1) about $z=0$ in powers of z and (2) about $z=1$ in powers of $z-1$. Also state the region of validity.

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$ using contour integration.

15. (a) (i) Using Laplace transform solve $y'' + 2y' - 3y = 3,\ y(0) = 4,\ y'(0) = -7$.

(ii) Find the Laplace transform of the periodic function defined by the triangular wave $f(t) = \begin{cases} \dfrac{t}{a}, & 0 \le t \le a \\ \dfrac{(2a - t)}{a}, & a \le t \le 2a \end{cases}$ and $f(t + 2a) = f(t)$.

Or

(b) (i) Using convolution, solve the initial value problem $y'' + 9y = \sin 3t,\ y(0) = 0,\ y'(0) = 0$.

(ii) Using Laplace transform, find the solution of $y' + 3y + 2\int_0^t y\,dt = t,\ y(0) = 0$.
