E 5083

M.E. DEGREE EXAMINATION, MAY/JUNE 2007.

First Semester

Applied Electronics

MA 151 — APPLIED MATHEMATICS FOR ELECTRONICS ENGINEERS

(Common to M.E. – Communication Systems, M.E. – Optical Communication, M.E. – Digital Communication and Network Engineering and M.E. – VLSI Design)

(Regulation 2002)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $u(x, t)$ is a function of x and t, prove that $L\left[\frac{\partial^2 u}{\partial t^2}\right] = s^2\overline{u} - su(x, 0) - u_t(x, 0)$.

2. Write down D'Alembert's solution of the initial value problem of Cauchy type $u_{tt} = c^2 u_{xx}, -\infty < x < \infty, t \geq 0$ subject to initial conditions $u(x, 0) = \eta(x)$, $u_t(x, 0) = \upsilon(x)$.

3. Write down the value of $J_{3/2}(x)$.

4. Prove that $\int J_1(x)dx = -J_0(x)$.

5. Prove that $P_n(1) = 1$.

6. A is known to hit the target in 2 out of 5 shots whereas B is known to hit the target in 3 out of 4 shots. Find the probability that the target being hit when they both try.

7. The moment generating function of a random variable X is given by $\mu_X(t) = \frac{1}{3}e^t + \frac{4}{15}e^{3t} + \frac{2}{15}e^{4t} + \frac{4}{15}e^{5t}$ find the probability density function of X.

8. The regression lines of two random variables X and Y is given by $3X + Y = 10$ and $3X + 4Y = 12$. Find the coefficient of correlation between X and Y.

9. State Little's formula.

10. What is the probability that a customer has to wait more that 15 minutes to get his service completed in $(\mu/\mu/1):(\infty/FIFO)$ queue system if $\lambda = 6$ per hour and $\mu = 10$ per hour.

PART B — (5 × 16 = 80 marks)

11. (a) Using Lapalce transform method, solve the initial boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \cos\omega t$, $0 \le x \le \infty$, $0 \le t \le \infty$ subject to the initial and boundary conditions $u(0,t) = 0$, $\frac{\partial u}{\partial t}(x,0) = 0$ $u(x,0) = 0$ and u is bounded as $x \to \infty$. (16)

Or

(b) A string is stretched and fixed between two points (0, 0) and $(l, 0)$. Motion is initiated by displacing the string in the form $u = \lambda \sin\frac{\pi x}{l}$ and released from rest at time $t = 0$. Find the displacement of any point on the string at time t. (16)

12. (a) (i) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. (8)

(ii) Express $J_5(x)$ interms of $J_0 x$ and $J_1(x)$. (8)

Or

(b) (i) Prove that $J_0(x) = \frac{1}{\pi}\int_0^{\pi}\cos(x\cos\phi)\,d\phi$. (8)

(ii) Derive Rodrigue's formula of Legendre's polynomial. (8)

13. (a) (i) Prove that $n\,p_n(x) = x\,p_n'(x) - p_{n-1}'(x)$. (8)

(ii) If $f(x) = \begin{cases} 0 & -1 < x < 0 \\ x & 0 < x < 1 \end{cases}$ show that

$$f(x) = \frac{1}{4}p_0(x) + \frac{1}{2}p_1(x) + \frac{5}{16}p_2(x) - \frac{3}{32}p_4(x) + \cdots \quad (8)$$

Or

(b) (i) Find the M.G.F. and rth moment for the distribution whose p.d.f. is $f(x) = ke^{-x}$, $0 \le x \le \infty$. Find also the standard deviation. (8)

(ii) The probability of a man hittting a target is 1/3. How many times must he fire so that the probability of hitting the target atleast once is more than 90%? (8)

14. (a) (i) The density function of a random variable X is $f(x) = \begin{cases} \dfrac{k}{1+x^2}, & -\infty < x < \infty \end{cases}$ determine k and evaluate $p(x \ge 0)$. (6)

(ii) The joint p.d.f of X and Y is given by $f(x, y) = \begin{cases} e^{-x}, & 0 < y < x < \infty \\ 0 & \text{elsewhere.} \end{cases}$

(1) Find the marginal p.d.f of X and Y and

(2) Determine the correlation coefficient between X and Y. (10)

Or

(b) (i) If $f(x, y) = \begin{cases} 6/5(x+y^2), & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$

Find the probability

(1) $P(.2 < x < .5)$ and

(2) $P(.4 < y < .6)$

(ii) If X and Y are two positive independent continuous random variables with p.d.f $f_1(x)$ and $f_2(y)$ respectively, find p.d.f. of $U = \dfrac{X}{Y}$.

15. (a) If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 minutes before the picture starts and if it takes exactly 1.5 min. to reach the correct seat after purchasing the ticket.

(i) Can he expect to be seated for the start of the picture?

(ii) What is the probability that he will be seated for the start the picture?

(iii) How early must be arrive in order to be 99% sure of being seated for the start of the picture? (16)

Or

(b) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes determine L_s, L_f, W_s and W_f.
