Reg. No. : | | | | | | | | | | |

J 3722

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2009.

Annual Pattern – First Year

Civil Engineering

MA 1 X 01 — ENGINEERING MATHEMATICS – I

(Regulation 2004)

(Common to all branches of B.E./B.Tech.)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Define similar matrices.

2. For the curve $ay^2 = x^3$, find the curvature at (a, a).

3. Solve $\frac{d^2y}{dx^2} + 4y = e^{-2x}$.

4. Solve $L\frac{di}{dt} + Ri = E$ with the initial condition when $t = 0, i = 1$. E is a constant.

5. Sketch roughly the region of integration for the double integral $\int_0^1 \int_0^x f(x, y)\, dy\, dx$.

6. If $\phi = \frac{1}{r}$, find $\nabla\phi$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

7. Define analytic function of a complex variable.

8. Evaluate $\int_C \frac{dz}{z+3}$ where C is the circle $|z| = 1$.

9. Evaluate $\int_0^\infty e^{-4t} \cos 3t \, dt$.

10. If $L[f(t)] = \phi(s)$, prove that $L[f(at)] = \frac{1}{a}\phi\left(\frac{s}{a}\right)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ and hence find A^{-1} and A^4. (8)

(ii) A rectangular box open at the top is to have a volume of 32 C.C. Find the dimensions of the box requiring the least amount of sheets for its construction. (8)

Or

(b) (i) Reduce the quadratic form $2xy + 2yz + 2zx$ to the canonical form by an orthogonal reduction and state its nature. (10)

(ii) Find the maximum and minimum of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. (6)

12. (a) (i) Solve $[(3x+2)^2 D^2 + 3(3x+2)D - 36]y = 3x^2 + 4x + 1$. (8)

(ii) The differential equation satisfied by a beam with a uniform loading w kg/m with one end fixed and the other end subject to a tensile force P is given by $EI\frac{d^2y}{dx^2} = Py - \frac{wx^2}{2}$. Find the equation to the elastic curve subject to the boundary conditions $y = 0$ at $x = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$. (8)

Or

(b) (i) Solve $(D^2 + 4D + 3)y = 2e^{-x}(x^2 + 2)$ where $D \equiv \frac{d}{dx}$. (8)

(ii) The charge q on the plate of a capacitor of an electric circuit with circuit elements L, R and C is given by $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E \sin wt$. If the circuit is tuned to resonance so that $LCw^2 = 1$ and $CR^2 < 4L$ and with the initial conditions $q = 0, \frac{dq}{dt} = 0$ at $t = 0$, show that $q = \frac{E}{Rw}\left[e^{\frac{-Rt}{2L}}\left\{\cos pt + \frac{R}{2LP}\sin pt\right\} - \cos wt\right]$ where $p^2 = \frac{1}{LC} - \frac{R^2}{4L}$. (8)

13. (a) (i) Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dydx$ and hence evaluate it. (8)

(ii) Prove that div (grad $(r^n)) = n(n+1)r^{n-2}$, if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Hence deduce that $div\ grad\left(\frac{1}{r}\right) = 0$. (8)

Or

(b) (i) Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ an $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)

(ii) Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (8)

14. (a) (i) If $f(z) = u + iv$ is a regular function of $z = x + iy$, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (10)

(ii) Using Cauchy's integral formula, evaluate

$\int_C \frac{z+4}{z^2 + 2z + 5} dz$ where C is the circle $|z + 1 - i| = 2$ (6)

Or

(b) (i) Find the bilinear transformation which maps the points $1, i, -1$ of z – plane into $i, 0, -i$ of the w plane. Also find the image of $|z| < 1$. (8)

(ii) By the method of contour integration, prove that $\int_0^\infty \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$. (8)

15. (a) (i) Find $L\left(\frac{1-\cos t}{t}\right)$ and $L^{-1}\left(\cot^{-1}\frac{s}{k}\right)$. (8)

(ii) Derive Laplace transform of a periodic function $f(t)$ with a period T. (8)

Or

(b) (i) State and prove initial value theorem and verify it for $f(t) = 1 + e^{-t}(\sin t + \cos t)$. (8)

(ii) Solve $y'(t) = t + \int_0^t y(t-u)\cos u \, du$ given that $y(0) = 4$. (8)