Reg. No. : | | | | | | | | | | | |

G 1395

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2009.

Second Semester

Civil Engineering

MA 132 — MATHEMATICS — II

(Common to all branches of BE/B.Tech except Information Technology)

Time : Three hours Maximum : 100 marks

Use of statistical tables permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Evaluate $\int_0^{\pi} \int_0^{\sin\theta} r \, dr \, d\theta$.

2. Write Beta and Gamma functions.

3. Define divergence and curl of a vector point function.

4. Show that $F = (x + 2y)i + (y + 3z)j + (x^2 - 2z)k$ is solenoidal.

5. State C.R. equation in Cartesian and polar coordinates.

6. Find the image of $0 \le x \le 2$ under the transformation $w = iz$.

7. Evaluate $\int_C \frac{dz}{(z+1)(z+2)}$ where C is $|z| = 1/2$.

8. Identify the nature of singularity of $f(z) = \frac{e^{z^2}}{z^3}$.

9. Define rank correlation and give the formula.

10. Explain null and alternative hypothesis in sampling theory.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{dx\ dy}{1+x^2+y^2}$. (8)

(ii) Find the volume of the tetrahedran bounded by $x = 0$, $y = 0$, $z = 0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (8)

Or

(b) (i) Change the order of integration in $\int_0^\infty \int_x^\infty e^{-x-y}\,dx\,dy$ and evaluate. (8)

(ii) Evaluate $\int_0^1 x^m \left(\log\left(\frac{1}{x}\right)\right)^n dx$ in terms of Gamma function. (8)

12. (a) (i) If $\nabla\varphi = (y^2 - 2xyz^3)i + (3 + 2xy - x^2z^3)j + (6z^3 - 3x^2yz^2)k$ find φ. (8)

(ii) Evaluate $\int_C \overline{F}\cdot d\overline{r}$ where $\overline{F} = 2yi + 3xj$ and C is the circle $x^2 + y^2 = 4$. (8)

Or

(b) (i) Find $f(r)$ such that $f(r)\overline{r}$ is both solenoidal and irrotational. (8)

(ii) Using Stoke's theorem evaluate $\int_C (xy\,dx + xy^2\,dy)$ taking C to be a square with vertices (1, 1), (1, –1), (–1, 1) and (–1, –1). (8)

13. (a) (i) Show that $f(z) = \log z$ is analytic to everywhere in the complex plane except at origin and find its derivative. (8)

(ii) Find the image of the region bounded by (0, 0), (1, 0), (1, 2), (0,2) by the transformation $w = (1+i)z + 2 - i$. Sketch the image. (8)

Or

(b) (i) Show that $V = \log(x^2 + y^2)$ is harmonic. Find a function u such that $u + iv$ is analytic. (8)

(ii) Find the bilinear transformation which maps the points $z = \infty, i, 0$ onto $w = 0, i, \infty$ respectively. (8)

14. (a) (i) Using Cauchy's integral formula evaluate $\int_C \frac{e^{5z}}{(z+i)^4} dz$ where C is $|z| = 3$. (8)

(ii) Find the Laurent expansion in $\frac{1}{z^2(1-z)}$ valid in the region (1) $0 < |z| < 1$ (2) $1 < |z| < 4$. (8)

Or

(b) (i) Identify and classify the singular points of $(z-3)\sin\left(\frac{1}{z+2}\right)$ and $\frac{z - \sin z}{z^3}$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$ by Contour Integration. (8)

15. (a) Obtain the lines of regression and the coefficient of correlation for the following data : (16)

x :	1	2	3	4	5	6	7
y :	9	8	10	12	11	13	14

Or

(b) (i) The means of two large samples of sizes 2000 and 1000 are 68.0 and 67.5 gm. Can the sample be regarded as drawn from the same population of S.D. 2.25 gm? (8)

(ii) The table below gives the number of aircraft accidents that occurred during the various days of a week. Test whether the accidents are uniformly distributed over the week. (8)

Days :	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
Accidents :	14	18	12	11	15	14

Note : χ^2 at 5% level for 5 d.f. is 11.07, 6 d.f. is 12.59, 7 d.f. is 14.07.