Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐

H 2294

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2009.

Third Semester

Civil Engineering

MA 231 — MATHEMATICS – III

(Common to all branches of B.E./B.Tech. except Fashion Technology, Industrial Bio Technology, Bio Medical, Textile Chemistry)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form the partial differential equation by eliminating arbitrary functions from $z = f_1(x) f_2(y)$.

2. Solve $\left(4D^2 - 4DD' + D'^2\right) z = 0$.

3. State Euler's formula for Fourier coefficients of a function defined in $(c, c + 2l)$.

4. Find the half range cosine series of $f(x) = \cos x$, $0 < x < \pi$.

5. Classify the following partial differential equation :

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \, \partial y} + 4\frac{\partial^2 u}{\partial y^2} - 12\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} + 7u = x^2 + y^2.$$

6. If an insulated rod of length 50 cm has its ends A and B are maintained at 0°C and 100°C respectively, find the steady state solution of rod.

7. Show that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}(u) du$, where $\bar{f}(s) = \int_0^\infty e^{-st} f(t)\, dt$.

8. Find the inverse Laplace transform of $\frac{1}{s(s+1)}$.

9. If $F_s(s)$ is the Fourier sine transform of $f(x)$, show that

$$\mathbf{F}_s[f(x)\cos ax] = \frac{1}{2}[F_s(s+a) + F_s(s-a)].$$

10. If $F(s)$ is the Fourier transform of $f(x)$, show that

$$\mathbf{F}\{e^{ibx/a} f(x/a)\} = aF(as+b),\ a > 0.$$

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the general solution of $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$. (8)

(ii) Find the singular solution of the equation $z = px + qy + \left(\frac{q}{p} - p\right)$. (8)

Or

(b) (i) Find the general solution of the equation $z^2(p^2 + q^2) = x + y$. (8)

(ii) Solve the equation $(D^2 - 2DD' + D'^2)z = x^2y^2e^{x+y}$. (8)

12. (a) (i) Find a Fourier series to represent $x - x^2,\ -\pi < x < \pi$. (8)

(ii) Expand $f(x) = x\sin x$ as a cosine series in $0 < x < \pi$ and show that

$$1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} - = \frac{\pi}{2}.$$ (8)

Or

(b) (i) Find the Fourier series of the function $f(x) = \begin{cases} k, & -1 < x < 0 \\ x, & 0 < x < 1 \end{cases}$. Hence find $1 - \frac{1}{2} + \frac{1}{3} - .....$. (8)

(ii) Find the first two harmonics in the Fourier series of $y = f(x)$ which is defined in the following table $(0, \pi)$. (8)

x :	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
y :	10	12	15	20	17	11	10

13. (a) A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge $y = 0$ is

$$u(x,0) = 200 \sin \frac{\pi x}{10}, \quad 0 < x < 10$$

while two long edges $x = 0$ and $x = 10$ as well as the other short edge are kept at 0°C, find the steady state temperature at any point of the plate.(16)

Or

(b) An insulated rod of length l its ends A and B are maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained so, find the temperature at a distance x from A at time t. (16)

14. (a) (i) Find the Laplace transform of the periodic function defined by

$$f(x) = \begin{cases} t, & 0 \le t \le 3 \\ 0, & 3 \le t \le 6, \end{cases} \quad f(t+6) = f(t).$$

(ii) Solve the initial value problem $y'' - 6y' + 9y = t^2 e^{3t}$, $y(0) = 2$, $y'(0) = 6$ using Laplace transform.

Or

(b) (i) Find $L\left[\frac{\cos t - \cos 2t}{t}\right]$ and $L\{\sin at \sin bt\}$. (8)

(ii) Find the inverse Laplace transform of $\frac{1}{(s^2+1)(s+2)}$ using convolution theorem. (8)

15. (a) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 - x^2, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$

Hence, evaluate $\int_0^\infty \left(\frac{\sin x - x \cos x}{x^3}\right) \cos \frac{x}{2} dx$. (16)

Or

(b) (i) Solve the integral equation $\int_0^\infty f(x) \cos \alpha x dx = e^{-\alpha} (\alpha > 0)$. (8)

(ii) Using Fourier cosine transforms, evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$. (8)
