Reg. No. :

J 3471

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2009.

First Semester

(Regulation 2004)

Civil Engineering

MA 1101 — MATHEMATICS — I

(Common to all branches of B.E./B.Tech.)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the sum and product of the eigenvalues of the matrix $A = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$.

2. Write down the matrix corresponding to the quadratic form

   $2x^2 + 2y^2 + 3z^2 + 2xy - 4xz - 4yz$.

3. Find the equation of the plane through (1, 2, 3) parallel to the plane $4x + 5y - 3z + 7 = 0$.

4. Find the equation of the sphere passing through the points (0, 0, 0) (1, 0, 0), (0, 1, 0) and (0, 0, 1).

5. Find the co-ordinates of the centre of curvature of the curve $y = x^2$ at the point (1, –1).

6. Find the radius of curvature at $x = \frac{\pi}{2}$ on the curve $y = 4\sin x$.

7. If $u = xe^y z$, where $y = 2x$, $z = \sin x$ find $\frac{du}{dx}$.

8. If $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

9. Solve $(x^2D^2 + xD)\,y = 0$.

10. Find the particular integral of $(D^2 + 1)\,y = \cosh 2x$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (8)

(ii) Using, Cayley Hamilton theorem find A^{-1} and A^4 for the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$. (8)

Or

(b) (i) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$. (4)

(ii) Diagonalise the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ by means of orthogonal transformation. (12)

12. (a) (i) Find the direction cosines of the lines AB and CD where A (1, 2, –4), B (2, 1, –3), C (4, 6, –1) and D (5, 7, 0). Hence, find acute angle between them. (6)

(ii) Find the equation of the plane passing through the point (1, 2, –1) and perpendicular to the planes $x + y - 2z = 5$, $3x - y + 4z = 12$. (10)

Or

(b) (i) Find the length and equation of shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \text{ and}$$

$$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$$ (8)

(ii) Prove that two spheres $x^2 + y^2 + z^2 - 2x - 4y - 4z = 0$ and $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$ touch each other and find the point of contact. (8)

13. (a) (i) Find the point on the parabola $y^2 = 4x$, at which radius of curvature is $4\sqrt{2}$. (6)

(ii) Find the equation of the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (10)

Or

(b) (i) Find the envelope of the family of curves $y = mx + \frac{3}{2m}$, where m is the parameter. (6)

(ii) Find the circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (10)

14. (a) (i) If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$. (8)

(ii) Investigate for the maxima and minima if any, of the function $f(x, y) = x^3 + y^3 - 3xy$. (8)

Or

(b) (i) Obtain Taylor series expansion of $e^x \log(1 + y)$ in powers of x and y upto terms of third degree. (8)

(ii) The temperature T at any point (x, y, z) in space is $T = cxyz^2$, where c is constant. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = 1$. (8)

15. (a) (i) Solve : $\frac{dx}{dt} + y = e^t$, $x - \frac{dy}{dt} = t$. (8)

(ii) Solve : $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin(\log x)$. (8)

Or

(b) (i) Solve : $(3D^2 + D - 14)\, y = 8e^{2x} + \cos 4x$. (8)

(ii) Solve, by using method of variation of parameters $\frac{d^2y}{dx^2} + 4y = \tan x$. (8)

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