Reg. No. : | | | | | | | | | | | |

J 3280

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2009.

Second Semester

Civil Engineering

MA 1151 — MATHEMATICS — II

(Common to all branches (except Food Technology/Rubber and Plastics))

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_1^b \int_1^a \frac{dxdy}{xy}$.

2. What is the meaning of $\iint_R x\,dy\,dz$?

3. What is the greatest rate of increase of $\phi = xyz^2$ at (1, 0, 3)?

4. If $r = xi + yj + zk$ and S is the surface of the sphere of unit radius, find $\iint_S \bar{r}.d\bar{s}$.

5. Show that an analytic function with constant real part is constant.

6. If $u + iv$ is analytic, show that $v - iu$ and $-v + iu$ are also analytic.

7. Find the Laurent expansion of $\frac{1}{z^3(1-z)}$ in $|z| > 1$.

8. Evaluate $\int_C \frac{dz}{\sin z}$ where C is $|z| = 1$.

9. Find the Laplace transform of $\sin t \sinh t$.

10. Find the Laplace transform of $\int_0^t te^{-t} dt$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} dx\, dy\, dz$. (8)

(ii) Evaluate by changing the order of integration, $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y\, dx\, dy$. (8)

Or

(b) (i) Find by double integration the area enclosed by the curve $x^{2/3} + y^{2/3} = a^{2/3}$. (8)

(ii) Evaluate $\int_0^a \int_y^a \frac{x dx dy}{x^2 + y^2}$ by changing to polar coordinates. (8)

12. (a) (i) Find the directional derivative of $xy^2 + yz^3$ at (2, −1, 1) in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at (−1, 2, 1). (8)

(ii) Find $f(r)$ if the vector $f(r)\,\bar{r}$ is both solenoidal and irrotational. (8)

Or

(b) (i) State Green's Theorem. (2)

(ii) Verify Gauss divergence theorem for $\overline{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ over the cube $x = 0$, $x = 1$, $y = 0$, $y = 1$ and $z = 0$, $z = 1$. (14)

13. (a) (i) If $w = f(z)$ is analytic then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f'(z)| = 0$. (8)

(ii) Show that $v = e^x(x \cos y - y \sin y)$ is harmonic and find u such that $u + iv$ is analytic. (8)

Or

(b) (i) Show that $\sinh z$ is an analytic function and find its derivative. (8)

(ii) Find the bilinear transform which maps $z = 0, -1, i$ onto $w = i, 0, \infty$. (8)

14. (a) (i) State and prove Cauchy's integral formula. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$, $(a > b > 0)$ using contour integration. (8)

Or

(b) (i) Evaluate $\int_C \frac{z\,dz}{(z-1)(z-2)^2}$ where C in the circle $|z-2| = \frac{1}{2}$ using Cauchy's Integral formula. (8)

(ii) Find the Laurent expansion of $f(z) = \frac{7z-2}{(z+1)z\,(z-2)}$ in $1 < |z+1| < 3$. (8)

15. (a) (i) Find the Laplace transform of $te^{-4t}\sin 3t$ and $\frac{e^{-at} - e^{-bt}}{t}$. (5)

(ii) Find the inverse Laplace transform of $\frac{se^{-s}}{(s-3)^5}$. (5)

(iii) Find the Laplace transform of $f(t) = \begin{cases} a\,\sin \omega t, & 0 \le t \le \pi/w \\ 0, & \pi/w \le t \le 2\pi/w \end{cases}$ with $f\left(t + \frac{2\pi}{w}\right) = f(t)$. (6)

Or

(b) (i) Using Convolution theorem, find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$. (8)

(ii) Using Laplace transform, solve $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 6t^2e^{-3t}$, given that $y(0) = y'(0) = 0$. (8)
