

6. Verify that $y = \cosh(\lambda x) \cosh(-\lambda at)$ is a solution of $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$
7. State Fourier integral theorem.
8. State convolution theorem for Fourier transforms.
9. Find $Z(f(n))$, where $f(n) = n$ for $n = 0, 1, 2, \dots$
10. State the initial value theorem in Z -transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the complete solution of $pqxy = z^2$. (8)
- (ii) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2zx$. (8)

Or

- (b) (i) Find a singular solution to the equation $z = px + qy + \sqrt{1 + p^2 + q^2}$. (8)
 - (ii) Solve the equation $(D^2 - D'^2)(z) = e^{x-y} \sin(x+2y)$. (8)
12. (a) (i) Find the Fourier series of $f(x) = x^2$ in $[0, 2\pi]$ and periodic with period 2π . Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. (8)
 - (ii) Find the half-range cosine series of $f(x) = \pi x - x^2$ in $(0, \pi)$. Deduce that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$. (8)

Or

- (b) (i) If a is not an integer, find the complex Fourier series of $f(x) = \cos ax$ in $(-\pi, \pi)$. (8)
- (ii) Compute the first two harmonics of the Fourier series of $f(x)$ given in the following table: (8)

$x:$	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x):$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

13. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$, find the displacement of any point on the string at a distance x from one end at any time t . (16)

Or

- (b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge $y = 0$ is given by $u = x$ for $0 \leq x \leq 5$ and $(10-x)$ for $5 \leq x \leq 10$ and the two long edges $x = 0, x = 10$ as well as the other short edges are kept at 0°C , find the temperature $u(x, y)$ at any point (x, y) of the plate in the steady state. (16)

14. (a) (i) Find the Fourier transform of $f(x)$ if

$$f(x) = \begin{cases} 1, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}. \text{ Hence deduce that } \int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}. \quad (8)$$

- (ii) Find the Fourier Cosine transform of $e^{-a^2x^2}$ for any $a > 0$ and hence prove that $e^{-x^2/2}$ is self-reciprocal under Fourier Cosine transform. (8)

Or

- (b) (i) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & \text{if } |x| < a \\ 0, & \text{if } |x| > a > 0. \end{cases}$

$$\text{Hence deduce that } \int_0^{\infty} \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}. \quad (8)$$

- (ii) Find $F_c(e^{-ax})$, $F_c\left(\frac{1}{1+x^2}\right)$ and $F_c\left(\frac{x}{1+x^2}\right)$ (Here F_c stands for Fourier Cosine transform). (8)

15. (a) (i) By using convolution theorem, prove that the inverse Z-transform of $\frac{z^2}{(z+a)(z+b)}$ is $\frac{(-1)^n}{b-a} \{b^{n+1} - a^{n+1}\}$. (6)
- (ii) By the method of Z-transform solve $y(n+2) + 6y(n+1) + 9y(n) = 2^n$ given that $y(0) = 0$ and $y(1) = 0$. (10)

Or

- (b) (i) Find the Z-transform of $\cos n\theta$ and hence find $Z(n \cos n\theta)$. (8)
- (ii) Solve the equation (using Z - transform) $y(n+2) - 5y(n+1) + 6y(n) = 36$ given that $y(0) = y(1) = 0$. (8)