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**Question Paper Code : S 4696**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Fourth Semester

Mechanical Engineering

MA 038 — NUMERICAL METHODS

(Common to Aeronautical Engineering, Civil Engineering, Instrumentation and Control Engineering, Automobile Engineering, Mechatronics Engineering and Production Engineering)

(Also common to Fifth Semester – Computer Science and Engineering, Metallurgical Engineering, Polymer Technology and Sixth Semester – Chemical Engineering, Electronics and Instrumentation Engineering, Textile Technology, Leather Technology)

(Regulation 2001)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define diagonal dominance of a system of algebraic equations.
2. How do you find an initial approximation of a root of an equation while solving using numerical methods?
3. Define operators  $\Delta$ ,  $\nabla$ ,  $\delta$  and  $\mu$ .
4. Show that  $E = e^{hD}$ , where  $E, D$  are shifting and differential operators respectively and  $h$  is interval length.
5. Explain the geometrical meaning of Trapezoidal rule.
6. What is the error in Simpson's one-third rule?
7. Mention any two single step methods for solving an ordinary differential equation subject to initial conditions.
8. Using Euler's method compute  $y(0.2)$  and  $y(0.4)$ , given  $y' = x + y$ ,  $y(0) = 1$ .

9. Classify the following partial differential equation.

$$x \frac{\partial^2 u}{\partial x^2} - 2x \frac{\partial^2 u}{\partial x \partial y} + (x-1) \frac{\partial^2 u}{\partial y^2} + 2 \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0.$$

10. Write finite differences of second order for 2D Laplace equation and obtain standard five point formula.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a real root of the equation  $\cos x = 3x - 1$  correct to 4 decimal places by iteration method.
- (ii) Describe Gauss-Seidel method for solving linear system of equations.

Or

- (b) (i) Solve the following system of equations using Gauss-elimination method. Use partial pivoting if necessary.

$$4x_1 + x_2 + x_3 = 4, \quad x_1 + 4x_2 - 2x_3 = 4 \quad \text{and} \quad 3x_1 + 2x_2 - 4x_3 = 6.$$

- (ii) Find the dominant eigenvalue and the corresponding eigenvector of

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \text{ using power method. Hence find the least eigen}$$

value of  $A^{-1}$ .

12. (a) (i) Using Stirling's formula and Bessel formula obtain the value of  $y(45)$  given.

$x:$	40	44	48	52
$y:$	51.08	63.24	70.88	79.84

- (ii) Derive Newton's forward and backward interpolation formulae.

Or

- (b) (i) The following data are taken from the steam table.

Temp. (°C):                    140    150    160    170    180

Pressure (kgf/cm<sup>2</sup>): 3.685   4.854   6.302   8.076   10.225

Find the pressure at temperatures 142° and 175°.

- (ii) Find the equation  $y = f(x)$  of least degree using Newton's divided difference formula which passes through the points  $(-1, -21)$ ,  $(1, 15)$ ,  $(2, 12)$ ,  $(3, 3)$ . Find also  $y$  at  $x = 0$ .

13. (a) (i) Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  at  $x = 0.96$  from the following data.

$x$ : 0.96    0.98    1.00    1.02    1.04

$y$ : 0.7825   0.7739   0.7651   0.7653   0.7473

- (ii) Evaluate  $\int_1^2 \frac{2x}{1+x^4} dx$ , using the Gauss-Legendre 2-point and 3-point quadrature rules.

Or

- (b) (i) The velocity  $v$  of a particle at distance  $s$  from a point on its path is given by the table :

$s$  (ft):        0   10   20   30   40   50   60

$v$  (ft/sec.):   47   58   64   65   61   52   38

Estimate the time taken to travel 60 ft. by using Simpson's 1/3 rule. Compare the result with Simpson's 3/8 rule.

- (ii) Using Trapezoidal and Simpson's rules, evaluate  $\int_2^{2.6} \int_4^{4.4} \frac{dx dy}{xy}$ .

14. (a) (i) Find by Taylor's series method, the values of  $y$  at  $x = 0.1$  and  $x = 0.2$  to five places of decimals from  $\frac{dy}{dx} = x^2y - 1$ ,  $y(0) = 1$ .

- (ii) Using Runge-Kutta method of fourth order, find  $y(0.8)$  correct to four decimal places if  $y' = y - x^2$ ,  $y(0.6) = 1.7379$ .

Or

- (b) (i) Using Milne's method find  $y(4.4)$  given  $5xy' + y^2 - 2 = 0$  given  $y(4) = 1$ ,  $y(4.1) = 1.0049$ ,  $y(4.2) = 1.0097$  and  $y(4.3) = 1.0143$ .

- (ii) Compute the value of  $y(0.2)$  using Runge-Kutta method of fourth order given  $y'' = -y$ ,  $y(0) = 1$ ,  $y'(0) = 0$ .

15. (a) (i) Solve the boundary value problem :

$$xy'' + y = 0, \quad y(1) = 1, \quad y(2) = 2,$$

using finite difference method by taking 4 intervals.

(ii) Solve the equation  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  subject to the conditions  $u(x, 0) = \sin \pi x$ ,  $0 \leq x \leq 1$ ;  $u(0, t) = u(1, t) = 0$  using Crank-Nicolson method. Carryout computations for two levels, taking  $h = 1/3$ ,  $k = 1/36$ .

Or

(b) (i) Evaluate the pivotal values of the equation  $\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$ , taking  $h = 1$  upto  $t = 1$ . The boundary conditions are  $u(0, t) = u(5, t) = 0$ ,  $\frac{\partial u}{\partial t} = 0$  and  $u(x, 0) = x^2(5 - x)$ .

(ii) Solve the equation  $\nabla^2 u = -10(x^2 + y^2 + 10)$  over the square mesh with sides  $x = y = 0$ ,  $x = y = 3$  with  $u = 0$  on the boundary and mesh length is 1.