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Question Paper Code : Q 2293

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Second Semester

Civil Engineering

MA 1151 — MATHEMATICS — II

(Common to All Branches of B.E./B.Tech./except Food Technology)

(Regulation 2004)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^{\pi/2} \int_0^{\pi/2} \sin(\theta + \varphi) d\theta \, d\varphi$.

2. Obtain the limits of the double integral $\iint_R f(x,y) dx\, dy$, where R is the region in the first quadrant bounded by $x = 1$, $y = 0$ and $y^2 = 4x$.

3. If $\vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$, find div $(curl\vec{F})$.

4. Use Gauss divergence theorem, prove that $\iint_S \vec{r} \cdot \hat{n} ds = 3V$, where V is the volume enclosed by the surface S.

5. Find, where the function $f(z) = \frac{z^2 - 4}{z^2 + 1}$ ceases to be analytic.

6. Define conformal mapping.

7. Evaluate $\int_{0}^{1+i} (x - y + ix^2)dz$ along the line joining $z = 0$ and $z = 1 + i$.

8. State Cauchy's theorem.

9. State and prove first shifting theorem.

10. Find $L^{-1}[\cot^{-1} s]$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\iint (x + y)dx\,dy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)

(ii) Using cylindrical coordinates, evaluate $\iiint z\,(x^2 + y^2 + z^2)dx\,dy\,dz$ through the volume of the cylinder $x^2 + y^2 = a^2$ intercepted by the planes $z = 0$ and $z = h$. (8)

Or

(b) (i) Evaluate $\iint r^2\,dr\,d\theta$, over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$. (8)

(ii) Find the volume of the tetrahedron bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)

12. (a) (i) Find a and b such that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at (1, –1, 2). (8)

(ii) Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (8)

Or

(b) (i) Prove that $div(grad\, r^n) = n(n+1)r^{n-2}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Hence deduce that $\nabla \cdot \left[\nabla\left(\frac{1}{r}\right)\right] = 0$. (8)

(ii) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential and the work done by $\vec{F}$ in moving an object in this field from (1, –2, 1) to (3, 1, 4). (8)

13. (a) (i) Show that the function $w = z^n$ where n is a positive integer, is analytic everywhere in the complex plane and find its derivative $\frac{dw}{dz}$. (8)

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. (8)

Or

(b) (i) If $f(z) = u + iv$ is analytic and $u - v = (x - y)(x^2 + 4xy + y^2)$, find $f(z)$ interms of z. (8)

(ii) Find the image of the circle $|z - 1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$. Show the result graphically (rough sketch). (8)

14. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{z+4}{z^2 + 2z + 5}\, dz$ where C is the circle $|z + 1 - i| = 2$. (8)

(ii) Find the Laurent's series expansion for $f(z) = \frac{7z - 2}{(z+1)z(z-2)}$ in $1 < |z + 1| < 3$. (8)

Or

(b) (i) By the method of contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$. (8)

(ii) Using the method of contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)}\, dx, \ (a, b > 0)$. (8)

15. (a) (i) Verify initial and final value theorems for the function $f(t) = 1 + e^{-t}(\sin t + \cos t)$. (8)

(ii) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ and $f(t + 2a) = f(t)$. (8)

Or

(b) (i) Using convolution theorem, find $L^{-1}\left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right]$. (8)

(ii) Using Laplace transform method, solve $\frac{d^2y}{dt^2} + 9y = 18t$ given that $y(0) = 0 = y\left(\frac{\pi}{2}\right)$. (8)
