Reg. No. : | | | | | | | | | | | |

Question Paper Code : P 1385

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Fourth Semester

Mechanical Engineering

MA 1253/MA 1259 — PROBABILITY AND STATISTICS

(Regulation 2004)

(Common to Automobile Engineering, Production Engineering, Information Technology, Textile Technology (Textile Chemistry), Textile Technology, and Textile Technology (Fashion Technology))

(Common to B.E. (Part-Time) Third Semester – Regulation 2005)

Time : Three hours

Maximum : 100 marks

Statistical tables are permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A sample space contains three sample points with associated probabilities given by $2p$, p^2 and $4p-1$. Find the value of p.

2. Two dice are rolled. What is the conditional probability that atleast one lands on 6 given that the dice land in different numbers.

3. Suppose the number of accidents occurring weekly on a particular stretch of a highway follows a Poisson distribution with mean 3. Calculate the probability that there is atleast one accident this week.

4. Find the probability density function of $Y = e^X$ if X is a uniform random variable over (0, 1).

5. Let X and Y be continuous random variables having joint density func
$f(x, y) = \frac{3}{2}(x^2 + y^2)$, $0 \le x \le 1$, $0 \le y \le 1$. Determine $P(X < 1/2, Y > 1/2)$.

6. The correlation coefficient of two random variables X and Y is $-1/4$ while th variances are 3 and 5. Find the covariance.

7. A random sample of 100 recorded deaths in the United States during the p year showed an average life span of 71.8 years. Assuming a populati standard deviation of 8.9 years, does this seem to indicate that the mean l span today is greater than 70 years? Use a 0.05 level of significance.

8. Define Type I and Type II errors.

9. Define Latin square Design.

10. Depict the ANOVA table for a two way classification.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) A random variable X has the density function

$$f(x) = \begin{cases} cx^2, & 1 \le x \le 2 \\ cx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find the constant c and $P(1/2 < X < 3/2)$. (6)

(ii) Find the first four moments about the origin for a random variable X having density function $f(x) = \frac{4x(9 - x^2)}{81}$, $0 \le x \le 3$. (10)

Or

(b) (i) Given $p(n) = \frac{2}{3^n}$, $n = 1, 2, 3, \ldots$ is probability function of a random variable X, determine the distribution function and hence find $P(X \ge 3)$. (4)

(ii) If a random variable X has the moment generating function $M_X(t) = \frac{2}{2-t}$, determine the variance of X. (4)

(iii) A computer center has three printers A, B and C, which prints at different speeds. Programs are routed to the first available printer. The probability that the programs are routed to the printers A, B and C are 0.6, 0.3 and 0.1 respectively. Occasionally a printer will jam and destroy a print out. The probability that printers A, B and C will jam are 0.01, 0.05 and 0.04 respectively. Your program is destroyed when a printer jams. What is the probability that printer A is involved? (8)

12. (a) (i) Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.7,

(1) What is the probability that the target would be hit on tenth attempt?

(2) What is the probability that it takes him less than 4 shots?

(3) What is the probability that it takes him an even number of shots? (8)

(ii) State and prove the memoryless property of an exponential distribution. (8)

Or

(b) (i) Trains arrive at a station at 15 minutes intervals starting at 4 a.m. If a passenger arrive at a station at a time that is uniformly distributed between 9.00 and 9.30, find the probability that he has to wait for the train for

(1) less than 6 minutes

(2) more than 10 minutes. (8)

(ii) In a class of students, the heights of the students is normally distributed. Six percent have height below 60 inches and 39 percent are between 60 and 70 inches. Find the mean and standard deviation of height. (8)

13. (a) The probability density function of two random variables X and Y is given by $f(x, y) = \frac{3}{2}(x^2 + y^2)$, $0 \le x \le 1$, $0 \le y \le 1$. Find the lines of regression of X on Y and Y on X. (16)

Or

(b) (i) The joint density function of two random variables X and Y is given by $f(x, y) = e^{-(x+y)}$, $x > 0, y > 0$. Find the probability density function of $U = \frac{X+Y}{2}$. (8)

(ii) If $X_1, X_2, ..., X_n$ are Poisson variates with mean 2, use Central limit Theorem to estimate $P(120 < S_n < 160)$ where $S_n = X_1 + X_2 + \cdots + \cdots + X_n$ and $n = 75$. (8)

14. (a) (i) The following are the number of sales which a sample of 9 salespeople of industrial chemicals in Gujarat and a sample of 6 sales people of industrial chemicals in Maharashtra made over a certain fixed period of time :

Gujarat :	59	68	44	71	63	46	69	54	48
Maharashtra :	50	36	62	52	70	41			

Assuming that the population sampled can be approximated closely with normal distributions having the same variance, test the null hypothesis $\mu_1 - \mu_2 = 0$ against the alternative hypothesis $\mu_1 - \mu_2 \neq 0$ at the 0.01 level of significance. (8)

(ii) When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows :

No. of mistakes in a page :	0	1	2	3	4	5	6
No. of pages :	275	72	30	7	5	2	1

Fit a Poisson distribution to the above data and test the goodness of fit at 5 percent level of significance. (8)

Or

(b) (i) Two independent samples of sizes 7 and 6 have the following values

Sample A :	28	30	32	33	31	29	34
Sample B :	29	30	30	24	27	28	

Examine whether the samples have been drawn from normal population having the same variance using 0.05 level of significance. (8)

(ii) To determine whether there really is a relationship between an employees performance in the company's training program and his or her ultimate success in the job, the company takes a sample of 400 cases from its very extensive files and obtains the results shown in the following table :

	Below average	Average	Above average
Poor	23	60	29
Average	28	79	60
Very good	9	49	63

Use the 0.01 level of significance to test the null hypothesis that the performance in the training program and success in the job are independent. (8)

15. (a) In order to determine whether there is significant difference in the durability of 3 makes of computer, samples of size 5 are selected from each make and the frequency of repair during the first year of purchase is observed. The results are as follows :

A	5	6	8	9	7
B	8	10	11	12	4
C	7	3	5	4	1

Test whether there is significant difference in the durability of the 3 makes of the computers. (16)

Or

(b) Four farmers each used four types of manures for the crop and obtained the yield (in quintals) as below :

Farmers	1	2	3	4
A	22	16	21	12
B	23	17	19	13
C	21	14	18	11
D	22	15	19	10

Is there any significant difference between

(i) farmers

(ii) manures? (16)