Reg. No. : | | | | | | | | | | | |

Question Paper Code : P 1388

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Fourth Semester

Biotechnology

MA 1255 — PROBABILITY AND STATISTICS

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Use of Statistical and Control Chart is permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Obtain the mean of Poisson distribution.

2. If the random variable X takes the values, 1, 2, 3 and 4 such that $2P(X=1)=3P(x=2)=P(X=3)=5P(x=4)$, find the probability distribution and cumulative distribution function of X.

3. If the joint pdf of (X, Y) is $f(x, y)=\frac{1}{4}$, $0 \le x, y \le z$, find $P(X+Y \le 1)$.

4. Prove that $Cov(X, Y)=E(XY)-E(X)\cdot E(Y)$.

5. Examine whether the poisson process $\{X(t)\}$ given by the probability law $P\{X(t)=r\}=\frac{e^{-\lambda t}(\lambda t)^r}{r!}$, $(r=0, 1, 2,...)$, is covariance stationary.

6. Find the mean of the population size in a linear birth and death process.

7. A one-year guarantee is given based on the assumption that no more than 10% of the items will be returned. Assuming an exponential distribution, what is the maximum failure rate that can be tolerated?

8. Discuss briefly the kinds of maintainability.

9. Explain completely randomised design.

10. What is control chart? Name the types of control charts.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) A and B alternately throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is $\frac{30}{61}$. (8)

(ii) The probability distribution of a R.V. X is given below :

X :	0	1	2	3
$P(X)$:	0.1	0.3	0.5	0.1

If $Y = X^2 + 2X$, find the probability distribution, mean and variance of Y. (8)

Or

(b) (i) If the life X (in years) of a certain type of car has a Weibull distribution with the parameter $\beta = 2$, find the value of the parameter α, given that probability of the life of the car exceeds 5 years is $e^{-0.25}$. For these values of α and β, obtain the mean and variance of X. (8)

(ii) For geometric distribution $p(x) = 2^{-x}$, $x = 1, 2, 3, \ldots$, prove that Chebychev's inequality gives $P[|X-2| \le 2] > \frac{1}{2}$ while the actual probability is $\frac{15}{16}$. (8)

12. (a) (i) Let (X, Y) be the two dimensional non-negative continuous random variable having the joint density.

$$f(x, y) = \begin{cases} 4xy\, e^{-(x^2+y^2)}, & x \geq 0,\ y > 0 \\ 0 & , \text{ otherwise} \end{cases}$$

Prove that the density function of $u = \sqrt{x^2 + y^2}$ is

$$h(u) = 2u^3 e^{-u^2}, \quad 0 \leq u < \infty$$
$$0 \quad , \quad \text{elsewhere}$$

(ii) Given the random variable X have the marginal density

$$f_1(x) = 1, \quad -\frac{1}{2} < x < \frac{1}{2}$$

and the conditional density of Y be

$$f(y/x) = \begin{cases} 1, & x < y < x+1, \quad -\frac{1}{2} < x < 0 \\ 1, & -x < y < 1-x, \quad 0 < x < \frac{1}{2} \end{cases}$$

Show that the variable X and Y are uncorrelated.

Or

(b) (i) The joint pdf of two R.Vs X and Y is given by

$$f(x, y) = \frac{9(1+x+y)}{2(1+x)^4\,(1+y)^4}, \quad 0 \leq x < \infty \ \&\ 0 < y < \infty$$

Find the marginal distribution of X and the conditional distribution of Y for $X = x$.

(ii) Obtain the regression equation of Y on X for the distribution $f(x, y) = \dfrac{y}{(1+x)^4} \exp\left(-\dfrac{y}{1+x}\right);\ x, y \geq 0$.

13. (a) (i) The process $\{X(t)\}$ whose probability distribution under certain conditions is given by

$$P\{X(t)=n\} = \frac{(at)^{n-1}}{(1+at)^{n+1}}, \quad n=1,2,\ldots$$
$$= \frac{at}{1+at}, \quad n=0$$

Show that it is not stationary. (8)

(ii) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if her drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability he takes a car on the third day and (3) the probability that he drives to work in the long run. (8)

Or

(b) (i) If customers arrive at a counter in accordance with a poisson process with a mean rate 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 minute (2) between 1 minute and 2 minute and (3) 4 minute or less. (8)

(ii) Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$, where A and B are RVs, is Weakly Stationary Process if (1) $E(A)=E(B)=0$ (2) $E(A^2)=E(B^2)$ and (3) $E(AB)=0$. (8)

14. (a) (i) A cutting tool wears out with a time to failure is normally distributed. It is known that about 34.5% of the tools fail before 9 working days and about 78.8% fail before 12 working days. (1) compute the MTTF (2) determine its design life for a reliability of 0.99 (3) determine the probability that the cutting tool will last one more day given that it has been in use for 5 days. (8)

(ii) A system consists of two sub-systems in parallel. The reliability of each sub-system is given by $R(t) = e^{-\left(\frac{t}{\theta}\right)^2}$. Assuming that common mode failure may be neglected, determine the system Mean time to failure (MTTF). (8)

Or

(b) (i) A time to repair a power generator is best described by its pdf

$$m(t) = \frac{t^2}{333}, \; 1 \le t \le 10 \text{ hours}$$

(1) Find the probability that a repair will be completed in 6 hours. (3)

(2) What is the MTTR? (3)

(3) Find the repair rate. (2)

(ii) The density function of the time to failure in years of the gizmos (for use on widgets) manufactured by a certain company is given by

$$f(t) = \frac{200}{(t+10)^3}, \quad t \ge 0.$$

(1) Derive the reliability function and determine the reliability for the first year of operation. (3)

(2) Compute the MTTF. (3)

(3) What is the design life for a reliability 0.95? (2)

15. (a) Five breeds of cattle B_1, B_2, B_3, B_4, B_5 were fed on four different rations R_1, R_2, R_3, R_4. Gains in weight in kg over a given period were recorded and given below :

	B_1	B_2	B_3	B_4	B_5
R_1	1.9	2.2	2.6	1.8	2.1
R_2	2.5	1.9	2.3	2.6	2.2
R_3	1.7	1.9	2.2	2.0	2.1
R_4	2.1	1.8	2.5	2.3	2.4

Is there a significant difference between (i) bread and (ii) rations?

Or

(b) 10 samples each of size 50 were inspected and the number of defectives in the inspection were : 2, 1, 1, 2, 3, 5, 5, 1, 2, 3. Draw the appropriate control chart for defectives.