Reg. No. :

L 5251

M.E. DEGREE EXAMINATION, MAY/JUNE 2009.

First Semester

Computer Aided Design

MA 148 — APPLIED MATHEMATICS FOR MECHANICAL ENGINEERS

(Common to M.E. Energy Engineering/M.E. Engineering Design/M.E. Thermal Engineering/M.E. Internal Combustion Engineering and M.E. Refrigeration and Air Conditioning)

(Regulation 2002)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace transform of $f(t) = t \sin at$.

2. Find the Fourier sine transform of $f(x)$ defined as

$$f(x) = \begin{cases} \sin x & \text{when } 0 < x < a \\ 0 & \text{when } x > a. \end{cases}$$

3. What is spherical mean?

4. Let $u(x, y)$ be the steady state temperature distribution in a long square bar of side π with one face maintained at constant temperature T and the other faces at zero temperature. Formulate above problem as Boundary Value Problem.

5. Show that straight line is the shortest distance between two point in plane.

6. Write the other two forms of Euler's formula.

7. What is the order of error in solving Laplace and Poisson equation?

8. Where do you apply the hyperbolic equation in mechanical engineering?

9. What is Schwarz Christoffel transformation?

10. Find the invariant point of the transformation $w = iz^2$.

PART B — (5 × 16 = 80 marks)

11. (a) Using the Laplace transform method, solve the PDE $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$, $u\left(\frac{\pi}{2}, t\right) = 0$, $\frac{\partial u(0,t)}{\partial x} = 0$, $u(x,0) = 30\cos 5x$. (16)

Or

(b) Using the Fourier cosine transform, find the temperature $u(x,t)$ in a semi infinite rod $0 \le x \le \infty$, determined by the PDE $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$ subject to $u(x,0) = 0$, $0 \le x \le \infty$, $\frac{\partial u(0,t)}{\partial x} = -u_0$ when $x = 0$ and $t > 0$, $u, \frac{\partial u}{\partial x}$ both tend to zero as $x \to \infty$. (16)

12. (a) Using the finite Fourier transform, solve the Laplace equation

$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$, $0 < x < \pi$, $0 < y < y_0$ subject to

$V(0,y) = 0$, $V(\pi, y) = 1$, $V_y(x,0) = 0$, $V(x, y_0) = 1$. (16)

Or

(b) (i) Drive the solution of Poisson equation. (8)

(ii) Explain about properties of harmonic function. (8)

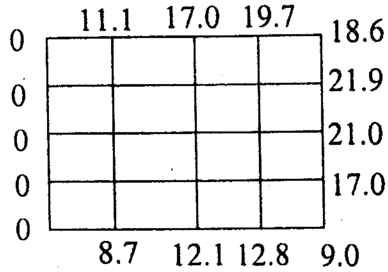
13. (a) (i) Find the extremal of the functional $V(y(x)) = \int_0^{\frac{\pi}{2}} \left(y^{11^2} - y^2 + x^2\right)dx$, satisfying the conditions. $y(0) = 1$, $y^1(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$, $y^1\left(\frac{\pi}{2}\right) = -1$. (8)

(ii) Prove that the sphere is the solid figure of revolution which for a given surface area has maximum volume (8)

Or

(b) Using Ritz method, solve the BVP : $y^{11} + y + x = 0$, $0 \le x \le 1$, $y(0) = 0$, $y(1) = 0$. (16)

(a) Determine by Liebmann's iteration method the values at the interior lattice points of a square region of the harmonic function $u(x,y)$ whose boundary values are given as shown in the figure below (16)



Or

(b) (i) Evaluate the pivotal values of the equation $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$ taking $h = 1$ upto $t = 1.25$. The boundary conditions are $u(0,t) = u(5,t) = 0$, $\frac{\partial u(x,0)}{\partial t} = 0$ and $u(x,0) = x^2(5 - x)$. (8)

(ii) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to conditions $u(x,0) = \sin \pi x$, $0 \le x \le 1$, $u(0,t) = u(1,t) = 0$ using Crank Nicolson method. (8)

15. (a) Use the Schwarz Christoffel transformation to show that $w = \log\left(2\sqrt{(z^2 + z)} + 2z + 1\right)$ maps the upper half plane conformally onto the interior of a semi infinite strip. (16)

Or

(b) Discuss the flow pattern associated with the complex potential

$$\Omega(z) = u_0\left(z + \frac{a^2}{z}\right) + \frac{i\gamma}{2\pi}\log z. \quad (16)$$