Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐

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M.C.A. DEGREE EXAMINATION, AUGUST/SEPTEMBER 2008.

Second Semester

DMC 1651 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. The product of two eigen values of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.

2. Show that of $A = \begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$ satisfy the Cayley – Hamilton Theorem.

3. If $S = \{a,b,p,q\}, Q = \{a,p,t\}$ find $S + Q, S - Q$.

4. Let $f : R \to R$ is given by $f(x) = x^2 - 2$. Find f^{-1}.

5. Define a functionally complete set.

6. Write CNF of $(P \rightleftharpoons Q)$.

7. Write the context free grammar for $L = \left\{ a^n b^n / n \geq 1 \right\}$.

8. Define a regular grammar.

9. Define a NFA with $\in$– closure.

10. Draw the state diagram of the automaton $M = (\{q_0, q_1, q_2, q_3\}, \{a,b\}, \delta, q_0, \{q_3\})$

	a	b
q_0	$\{q_0\ q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the system of equations

$x + y + z = 3, x + y - z = 1, 3x + 3y - 5z = 1$. (8)

(ii) Using the Cayley – Hamilton theorem find A^{-1}, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. (8)

Or

(b) (i) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. (8)

(ii) Test the consistency of the system of equations and if consistent, solve.

$2x - y - z = 2$; $x + 2y + z = 2$;
$4x - 7y - 5z = 2$. (8)

12. (a) (i) Show that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$. (8)

(ii) Prove that the relation "congruence modulo m" over the set of positive integers is an equivalence relation. (8)

Or

(b) (i) In how many ways can 4 mathematics books, 3 history books, 5 chemistry books and 7 biology books are arranged on a self so that all books of the same subjects are together. (8)

(ii) Let $f : R \to R, g : R \to R, h : R \to R$ be three functions where $f(x) = x^3 - 7$, $g(x) = x + 25$, and $h(x) = x^{1/3}$. Find $g \circ f, f \circ f$ and $f \circ g \circ h$. (8)

13. (a) (i) Obtain the *pcnf* and *pdnf* of the formula $(\neg P \to R) \wedge (Q \to P)$. (8)

(ii) Use the rules of inference obtain the implication,

$(\forall x)(P(x) \to Q(x)), (\forall x)(R(x) \to \neg Q(x)) \Rightarrow (\forall x)\ (R(x) \to \neg P(x))$. (8)

Or

(b) (i) Check the tautology of $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$.

(ii) Find the consistency of the following. (8)

If war is near, then the army would be mobilized. If the army is mobilized then labour costs are high. However the war is near and yet labour costs are not high. (8)

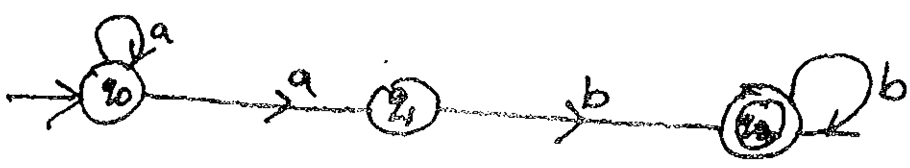
14. (a) (i) Construct a grammar for the language $L = \{ a^i b^{2i} / i \geq 1 \}$. (8)

(ii) Using the pumping lemma for regular language, Show that $\{a^n b^n / n \geq 0\}$ is not regular. (8)

Or

(b) (i) Construct a FSM to recognise whether or not a string over the alphabet $\{0, 1\}$ contains an even number of 1's.

(ii) Construct a DFA equivalent to the NFA (8)



15. (a) (i) Give the algorithm to convert a NFA to DFA. (8)

(ii) Is the language $L = \{ a^p / p \text{ is a prime number} \}$ regular. (8)

Or

(b) (i) Prove that if L is a regular language then there-exists a NFA to accept L. (8)

(ii) Construct an automata over $\{0, 1\}$, to accept the set of all strings which has consecutive three zeros. (8)

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