Reg. No. : | | | | | | | | | | | |

L 5248

M.E. DEGREE EXAMINATION, MAY/JUNE 2009.

First Semester

Structural Engineering

MA 145 — APPLIED MATHEMATICS

(Regulation 2002)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace transform of t^n, where n is a positive integer.

2. Find $L^{-1}\left[\dfrac{S+2}{S^2-4S+13}\right]$.

3. Find the Fourier sine series of e^{-ax}.

4. State Mean Value theorem for Harmonic functions.

5. Write down any two applications of calculus of variation.

6. What is Simplest Variational problem?

7. Let X and Y be two random variables such that $Y \le X$, then prove that $E(Y) \le E(X)$.

8. If a random variable X has the moment generating function $M_X(t) = \dfrac{2}{2-t}$, determine the variance of X.

9. If $\hat{\theta}$ is an unbiased estimate of θ, show that $\hat{\theta}^2$ is a biased estimator of θ^2.

10. Prove that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is consistent estimator of μ.

PART B — ($5 \times 16 = 80$ marks)

11. (a) Solve the IBVP described by

PDE : $u_{tt} = u_{xx}$, $0 < x < 1$, $t > 0$

BCs : $u(0, t) = u(1, t) = 0$, $t > 0$

ICs : $u(x, 0) = \sin \pi x$

$u_t(x, 0) = -\sin \pi x$, $0 < x < 1$ (16)

Or

(b) Solve the heat conduction problem described by

PDE : $K u_{xx} = u_t$, $0 < x < \infty$, $t > 0$

BC : $u(0, t) = u_0$, $t \geq 0$

IC : $u(x, 0) = 0$, $0 < x < \infty$

u and u_x both tend to zero as $x \to \infty$. (16)

12. (a) Find the steady-state temperature distribution $u(x, y)$ in a long square bar of side π with one face maintained at constant temperature u_0 and the other faces at zero temperature. (16)

Or

(b) Solve $u_{xx} + u_{yy} = 0$, $y \geq 0$ subject to the boundary conditions

$u(x, 0) = f(x)$, $-\infty < x < \infty$ and

$u(x, y) \to 0$, as $y \to \infty$. (16)

13. (a) Prove that the sphere is the solid figure of revolution which for a given surface area has maximum volume. (16)

Or

(b) (i) Solve $y'' + xy = -x$, $y(0) = y(1) = 0$ using Ritz method. (8)

(ii) Solve $\int_0^1 (1 + y''^2)\, dx$;

$y(0) = 0$, $y'(0) = 1$, $y(1) = 1$, $y'(1) = 1$. (8)

14. (a) (i) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution. (8)

(ii) Derive the mean and variance of the Gamma distribution. (8)

Or

(b) (i) In a partially destroyed record the following data are legible :

Variance of $X = 25$. Regression equation of X on Y is $5X - Y = 22$ and the regression equation of Y on X is $64X - 45Y = 25$. Find the correlation coefficient between X and Y. Also find regression coefficients. (8)

(ii) If X, Y denote the deviations of the variate from the A.M. and if $r = 0.5$, $\Sigma xy = 120$, $\sigma_y = 8$, $\Sigma x^2 = 90$ then find n. (8)

15. (a) (i) Is it possible to get the following from a set of experimental data? $r_{12} = 0.6, r_{23} = 0.8, r_{13} = -0.5$. (8)

(ii) If r_{12} and r_{13} are given, show that r_{23} must lie in the range $r_{12} \cdot r_{13} \pm \left(1 - r_{12}^2 - r_{13}^2 + r_{12}^2 r_{13}^2\right)^{1/2}$. (8)

Or

(b) (i) Find the M.L.Es of α and β for the random sample from the exponential population $f(x;\alpha,\beta) = y_0 e^{-\beta(x-\alpha)}, \alpha \le x \le \infty, \beta > 0$. y_0 being a constant. (8)

(ii) For the Poisson parameter θ, show that $\frac{1}{\bar{x}}$ is a consistent estimator of $1/\theta$. (8)
