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Question Paper Code : P 1384

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Fourth Semester

Computer Science and Engineering

MA 1252 — PROBABILITY AND QUEUEING THEORY

(Regulation 2004)

(Common to B.E. (Part-Time) Third Semester – Regulation 2005)

Time : Three hours Maximum : 100 marks

Use of statistical table is permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A lot of semiconductor chips contains 20 that are defective. Two are selected, at random, without replacement from the lot what is the probability that the second one selected is defective given that the first one was defective?

2. If the range of X is the set $\{0,1,2,3,4\}$ and $P[X=x]=0.2$, determine the mean and variance of the random variable.

3. The probability of a successful optical alignment in the assembly of an optical data storage product is 0.8. Assume the trials are independent, what is the probability that the first successful alignment requires exactly four trials?

4. Suppose X has an exponential distribution with mean equal to 10. Determine the value of x such that $P[X<x]=0.95$.

5. Determine the value of C such that the function $f(x,y)=c\,x\,y$ and $0<x<3$ and $0<y<3$ satisfies the properties of a joint probability density function.

6. Define covariance and correlation between the random variables X and Y.

7. Consider the random process $\{X(t), X(t) = \cos(t + \phi)\}$ where ϕ is uniform in $(-\pi/2, \pi/2)$. Check whether the process is stationary.

8. The one-step transition probability matrix of a Markov chain with states (0,1) is given by $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Is it irreducible Markov chain?

9. Define effective arrival rate with respect to an $(M \mid M \mid 1):(G_D / N / \infty)$ queueing model.

10. Write Pollaczek-Khintchine formula for the case when service time distribution is Erlang distribution with K phases.

PART B — (5 × 16 = 80 marks)

11. (a) Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful and 25% have been poor products.

(i) What is the probability that a product attains a good review? (6)

(ii) If a new design attains a good review, what is the probability that it will be a highly successful product? (5)

(iii) If a product does not attain a good review, what is the probability that it will be a highly successful product? (5)

Or

(b) (i) Obtain the moment generating function of the random variable X having probability density function $f(x) = \begin{cases} x, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$. (8)

(ii) A fair coin is tossed three times. Let X be the number of tails appearing. Find the probability distribution of X. And also calculate E(X). (8)

12. (a) (i) Derive the mean and variance of a Binomial random variable with parameters n and p. (10)

(ii) Suppose that X is a negative binomial random variable with $p = 0.2$ and $r = 4$. Determine the mean of X. (6)

Or

(b) (i) The time between process problems in a manufacturing line is exponentially distributed with a mean of 30 days. What is the expected time until the fourth problem? (4)

(ii) Find the moment generating function of a Normal random variable with parameters μ and σ and hence obtain its mean and standard deviation. (12)

13. (a) Determine the value of C that makes the function $F(x,y)=C(x+y)$ a joint probability density function over the range $0<x<3$ and $x<y<x+2$. Also determine the following.

(i) $P(X<1,\ Y<2)$ (8)

(ii) $P(Y>2)$ (4)

(iii) $E[X]$ (4)

Or

(b) (i) A fair coin is tossed 10 times. Find the probability of getting 3 or 4 or 5 heads using central limit theorem. (6)

(ii) If the joint probability density function of X and Y is

$$f(x,y)=\begin{cases} e^{-(x+y)} & \text{for } x>0,\ y>0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability density function of $Z=\dfrac{X}{X+Y}$. (10)

14. (a) Show that the random process $X(t)=A\sin(\omega t+\theta)$ is wide-sense stationary process where A and ω are constants and θ is uniformly distributed in $(0,\ 2\pi)$.

Or

(b) Define Poisson process and obtain the probability distribution for that. Also find the auto correlation function for the process.

15. (a) (i) For the $(M \mid M \mid 1):(G_D/\infty/\infty)$, derive the expression for L_q. (6)

(ii) Patients arrive at a clinic according to Poission distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(1) What is the probability that an arriving patient does not have to wait?

(2) What is the expected waiting time until a patient is discharged from the clinic?

Or

(b) Derive the Pollaczek-Khistchine formula for the $M|G|1$ queueing model.
