Reg. No. : | | | | | | | | | | |

Question Paper Code : P 1387

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Fourth Semester

Electronics and Communication Engineering

MA 1254 — RANDOM PROCESSES

(Regulation 2004)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — $(10 \times 2 = 20$ marks)

1. Two unbiased dice are thrown and the difference between the number of spots turned up is noted. Find the probability that the difference between the number is 4.

2. If the cdf of a RV is given by $f(x) = \begin{cases} 0, & \text{for } x < 0 \\ \frac{x^2}{16}, & \text{for } 0 \le x \le 4 \\ 1 & \text{for } 4 < x \end{cases}$ and find $P(x > 1/x < 3)$.

3. The probability that a candidate can pass in an exam is 0.6.

   (a) What is the probability that he pass in the third trial?

   (b) What is the probability that he pass before the 3rd trial?

4. Prove that the maximum value of the variance of the binomial distribution is $n/4$.

5. If $f(x,y) = e^{-(x+y)}$, $x \ge 0$; $y \ge 0$ is the joint pdf of x and y, find $P(x < 1)$.

6. If $f(x,y) = 8xy$, $0 < x < 1$, $0 < y < x$ is the joint pdf of x and y, find $f(y/x)$.

7. Determine whether the given matrix is irreducible or not $P = \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8 \end{pmatrix}$.

8. Show that the random process $X(t) = A\cos(w_0 t + \theta)$ is not stationary, if A and w_0 are constants and θ is uniformly distributed in $(0, \pi)$.

9. Find the power spectral density of a stationary process whose auto correlation function is $e^{-|\tau|}$.

10. Find the auto correlation function of a stationary process whose power spectral density function is given by $s(w) = \begin{cases} w^2 & \text{for } |w| \le 1 \\ 0 & \text{for } |w| > 1 \end{cases}$.

PART B — (5 × 16 = 80 marks)

11. (a) If $P(X = x) = \begin{cases} Kx, & x = 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$ represents a p.m.f.

(i) Find 'K'.

(ii) Find $P(x$ being a prime number).

(iii) Find $P\left\{\frac{1}{2} < x < \frac{5}{2} / x > 1\right\}$.

(iv) Find the distribution function.

Or

(b) The probability mass function of a Rv x is given by $P(i) = \frac{c\lambda^i}{i!} (i = 0,1,2 \ldots)$ where $\lambda > 0$ Find

(i) $P(x = 0)$

(ii) $P(x > 2)$.

12. (a) The time (in hours) required to repairs a machine is exponential, distributed with parameter $\lambda = 1/2$.

(i) What is the probability that the repair time exceeds 2 hours?

(ii) What is the conditional probability that a repair takes atleast 10h given that its duration exceeds 9h?

Or

(b) (i) If the life time X (in years) of a certain type of car has a Weibull distribution with parameter $\beta = 2$. Find the value of the parameter α given that probability that the life of the car exceeds 5 years is $e^{-0.25}$. For this values of α and β find the mean and variance of X.

(ii) An irregular 6 faced dice is such that the probability that it gives 3 odd numbers in 7 throws is twice the probability that it gives 4 odd numbers in 7 throws. How many sets of exactly 7 trials can be expected to give no odd number out of 5000 sets?

13. (a) If two dimensional random variables X and Y are uniformly distributed in $0 \le x \le y < 1$ find

(i) Correlation coefficient r_{xy}.

(ii) Regression equations.

Or

(b) If the joint pdf of a two dimensional $R_v\ (x, y)$ is given by

$$f(x, y) = \begin{cases} k\ (6 - x - y) \\ \quad 0 < x < 2 \\ \quad 2 < y < 4 \\ 0, \text{ elsewhere} \end{cases}$$

Find :

(i) the value of k

(ii) $p\,(x < 1,\ y < 3)$

(iii) $p\,(x < 1 / y < 3)$

(iv) $p\,(y < 3 / x < 1)$

(v) $p\,(x + y < 3)$.

14. (a) (i) If $x(t) = A \sin\ (wt + \theta)$, where A and W are constants and θ is RV uniformly distributed over $(-\pi, \pi)$. Find the auto correlation of $\{y(t)\}$, where $y(t) = x^2(t)$. $f(\theta) = \begin{cases} \frac{1}{2\pi}, & -\pi < \theta < \pi \\ 0 & \text{, otherwise.} \end{cases}$ (8)

(ii) For a Random process $x(t) = y \sin wt$, y is an uniformly distributed random variable in the interval $(-1, 1)$. Check whether the process is wide sense stationary or not. (8)

Or