B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Seventh Semester

Information Technology

MA 1256 — DISCRETE MATHEMATICS

(Regulation 2004)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Construct the truth table for the formula $\urcorner(P \wedge Q) \leftrightarrow (\urcorner P \vee \urcorner Q)$.

2. Define principal conjunctive normal form.

3. Write the negation of the statement, "Every city in Canada is clean".

4. Show that $(x)(H(x) \rightarrow M(x)) \wedge H(s) \Rightarrow M(s)$.

5. If $X = \{1, 2, 3, 4\}$ and $R = \{(x, y)/x > y\}$, find the relation matrix of R.

6. Define a Lattice.

7. For any set A, show that $\sim\sim A = A$.

8. Define a recursive relation.

9. Define a semigroup with an example.

10. If $f : G_1 \rightarrow G_2$ is a homomorphism of groups, show that Ker f is a normal subgroup of G_1.

11. (a) (i) Show that

$$\left((P \vee Q) \wedge \neg\left(\neg P \wedge \left(\neg Q \vee \neg R\right)\right)\right) \vee \left(\neg P \wedge \neg Q\right) \vee \left(\neg P \wedge \neg R\right) \text{ is a tautology.}$$

(ii) Without using truth table, show that
$$\neg(P \wedge Q) \to \left(\neg P \vee \left(\neg P \vee Q\right)\right) \Leftrightarrow \left(\neg P \vee Q\right).$$

Or

(b) (i) Obtain the principal disjunctive normal form of
$$P \to \left((P \to Q) \wedge \neg\left(\neg Q \vee \neg P\right)\right).$$

(ii) Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q.

12. (a) (i) Show that
$$(x)(P(x) \to Q(x)) \wedge (x)(Q(x) \to R(x)) \Rightarrow (x)(P(x) \to R(x)).$$

(ii) Show that the conclusion $(x)\left(F(x) \to \neg S(x)\right)$ follows from
$$(\exists x)(F(x) \wedge S(x)) \to (y)(M(y) \to W(y)) \text{ and } (\exists y)\left(M(y) \wedge \neg W(y)\right).$$

Or

(b) (i) Show that, $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

(ii) State the rules of UG, EG, US and ES.

13. (a) (i) Let $R = \{(1,2), (3,4), (2,2)\}$ and $S = \{(4,2), (2,5), (3,1), (1,3)\}$, find $R \circ S$, $S \circ R$, $R \circ (S \circ R)$, $(R \circ S) \circ R$, $R \circ R$ and $S \circ S$.

(ii) For any a, b, c in a lattice $(L, \leq)$, show that $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$.

Or

(b) (i) Let $X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(X, \leq)$.

(ii) In a Boolean algebra, show that
$$(a * b)' = a' \oplus b' \text{ and}$$
$$(a \oplus b)' = a' * b'.$$

14. (a) (i) Let $f(x) = x + 2$, $g(x) = x - 2$ and $h(x) = 3x$, for $x \in \mathbb{R}$, the set of real numbers. Find $g \circ f$, $f \circ g$, $f \circ f$, $g \circ g$ and $f \circ h \circ g$.

   (ii) Show that for any three sets A, B, C, $\chi_{A \cap (B \cup C)} = \chi_{(A \cap B) \cup (A \cap C)}$.

<div align="center">Or</div>

   (b) (i) Let $f : X \to Y$ and $g : Y \to X$. Prove that the function g is equal to f^{-1} only if $g \circ f = I_X$ and $f \circ g = I_Y$.

   (ii) Show that the set of divisors of a positive integer n is recursive.

15. (a) State and prove Lagrange's theorem on finite groups.

<div align="center">Or</div>

   (b) Prove that a code can correct all combinations of k or fewer errors if and only if the minimum distance between any two code words is atleast $2k + 1$.