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M.E. DEGREE EXAMINATIONS: DECEMBER 2009

First Semester

ENERGY ENGINEERING

MAT504: Applied Mathematics for Energy Engineering

Time: Three Hours **Maximum Marks: 100**

Answer All the Questions:-

PART A (10 x 2 = 20 Marks)

1. Define Pseudo inverse of a matrix and state two of its properties.
2. Find the generalized eigen vector of rank 2 corresponding to $\lambda = 5$ for the matrix

$$A = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

3. Write the Euler's equation for the functional $J\left[y(x)\right] = \int_a^b F(x, y, y', y'')\,dx$.
4. State the fundamental lemma of the calculus of variations.
5. Distinguish between pure and mixed integer programming problem. Mention two methods of solving an integer programming problem.
6. Find all basic solutions of the L.P.P:

$$\text{Max} \quad z = x_1 + x_2 + x_3$$

$$\text{Subject to} \quad x_1 + 2x_2 + x_3 = 4,$$

$$2x_1 + x_2 + 5x_3 = 5$$

$$x_1, x_2, x_3 \geq 0$$

7. State any four applications of Dynamic programming.
8. State Bellman's principle of optimality.
9. Define stationary random process.
10. Write any four properties of Power density spectrum.

PART B (5 x 16 = 80 Marks)

11. (a) (i) Find a matrix in Jordan canonical form that is similar to the matrix (8)

$$A = \begin{pmatrix} 5 & 2 & 2 \\ 3 & 6 & 3 \\ 6 & 6 & 9 \end{pmatrix}$$

(ii) Construct QR decomposition for the matrix. (8)

$$A = \begin{pmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{pmatrix}$$

(OR)

(b) (i) Determine the number of generalized eigen vectors of each rank corresponding to λ=4 that will appear in a canonical basis for $A = \begin{bmatrix} 4 & 2 & 1 & 0 & 0 & 0 \\ 0 & 4 & -1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix}$ (10)

(ii) Determine the chain that is generated by the generalized eigen vector of rank 2 for the given matrix $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ -1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ (6)

12. (a) (i) Find the maximum value of $z = 5x_1 + 7x_2$ subject to $x_1 + x_2 \le 4$, $3x_1 + 8x_2 \le 24$, $10x_1 + 7x_2 \le 35$, $x_1, x_2 \ge 0$. Using Simplex method. (8)

(ii) The owner of a small machine shop has four machinists available to assign to jobs for the day. Five jobs are offered with the expected profit in rupees for each machinist on each job being as follows: (8)

		Jobs				
		A	B	C	D	E
Machinists	1	6.2	7.8	5	10.1	8.2
	2	7.1	8.4	6.1	7.3	5.9
	3	8.7	9.2	11.1	7.1	8.1
	4	4.8	6.4	8.7	7.7	8.0

Find the assignment of a machinists to jobs that will result in a max. profit. Which job should be declined?

(OR)

(b) Solve the following transportation problem.

2	3	11	7	6
1	0	6	1	1
5	8	15	9	10
7	5	3	2	

13. (a) Use dynamic programming to solve

Max $Z = y_1 y_2 y_3$ subject to the constraints $y_1 + y_2 + y_3 = 5,\ y_1, y_2, y_3 \geq 0$.

(OR)

(b) Solve the following LPP by dynamic programming:

Maximize $z = 8x_1 + 7x_2$ subject to

$2x_1 + x_2 \leq 8$

$5x_1 + 2x_2 \leq 15,\ x_1, x_2 \geq 0$

14. (a) (i) Find the extremal of the functional $v\left[y(x)\right] = \int_0^1 \left(y^2 + x^2 y^1\right) dx$;

$y(0) = 0$; $y(1) = a$. (8)

(ii) Derive the differential equation of tree vibrations of a string. (8)

(OR)

(b) (i) Prove that the sphere is the solid figure of revolution which, for a given area, had maximum volume. (8)

(ii) Solve the boundary value problem $y'' + y + x = 0$. $y(0) = y(1) = 0$ by Rayleigh-Ritz method. (8)

15. (a) (i) Prove that $R_{XY}(\tau) = R_{YX}(-\tau)$ where R_{XY} is the cross correlation function. (8)

(ii) Show that a simple random walk $X(n)$ is a Markov chain. (8)

(OR)

(b) (i) If $x(t) = \beta \cos(50t + \phi)$ where β and ϕ are independent random variables with mean 0 and variance 1, ϕ is uniformly distributed in the interval $(-\pi, \pi)$. Show that $x(t)$ is w.s.s. (8)

(ii) Find the auto correlation function of a periodic time function $x(t) = A \sin \omega_0 t$. (8)

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