Q 9315

B.Sc. DEGREE EXAMINATION, MAY/JUNE 2006.

First Semester

Computer Technology/Information Technology/Apparel and Fashion Technology

BCS 112 — TRIGONOMETRY, ALGEBRA AND CALCULUS

(Regulations 2003)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $\frac{\sin\theta}{\theta} = \frac{5045}{5046}$, prove that θ is $1°58'$ nearly.

2. Separate the real and imaginary parts of $\sec(\alpha + i\beta)$.

3. Do the equations $x - 3y - 8z = 0$, $3x + y = 0$ and $2x + 5y = 6z = 0$ have a non-trivial solution? Why?

4. The product of two eigen values of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.

5. If $z = e^{x^2+y^2}$, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z\log z$.

6. State any two properties of Jacobians.

7. Evaluate $\int_0^{\pi} \sin^8 x\, dx$.

8. Find the area bounded by one arch of the curve $y = \sin a x$ and the x – axis.

9. Solve for x from the equations $x' - y = t$ and $x + y' = 1$.

10. Find the particular integral of $(D^2 + 4D + 4)y = x e^{-2x}$.

PART B — (5 × 16 = 80 marks)

11. (i) If $u_n = \int_0^{\pi/2} x^n \sin x \, dx$, show that $u_n + n(n-1)u_{n-2} = n \cdot \left(\frac{\pi}{2}\right)^{n-1}$. Hence deduce that $u_3 = \frac{3\pi^2}{4} - 6$. (8)

(ii) Find the area of a loop of the curve $a^2y^2 = x^2(a^2 - x^2)$. (8)

12. (a) (i) Prove that $\cos 7\theta = \cos^7\theta - 21\cos^5\theta\sin^2\theta + 34\cos^3\theta\sin^4\theta - 7\cos\theta\sin^6\theta$. (8)

(ii) If $\tanh\left(\frac{u}{2}\right) = \tan\left(\frac{\theta}{2}\right)$, show that $u = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$. (8)

Or

(b) (i) Prove that $64\sin^4\theta\cos^3\theta = \cos 7\theta - \cos 5\theta - 3\cos 3\theta + \cos\theta$. (8)

(ii) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\left(\frac{b}{a}\right)$ and hence evaluate $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$. (8)

13. (a) (i) Prove that the equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = 1$ are consistent and solve the same. (8)

(ii) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$. (8)

Or

(b) (i) Reduce the quadratic form below to its normal form by an orthogonal reduction

$q = 3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$. (10)

(ii) Using Cayley–Hamilton theorem, find the inverse of the matrix $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$. (6)

14. (a) (i) If $u = (x^2 + y^2 + z^2)^{-1/2}$, prove that $\sum \frac{\partial^2 u}{\partial x^2} = 0$. (8)

(ii) Discuss the maxima and minima of $x^3 + y^3 - 3x - 12y + 20$. (8)

Or

(b) (i) Use Taylor's formula to expand the function $x^2 + xy + y^2$ in powers of $(x - 1)$ and $(y - 2)$. (8)

(ii) If $u = xyz$, $v = xy + yz + zx$, $w = x + y + z$, prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = (x - y)(y - z)(z - x)$. (8)

15. (a) (i) Solve : $(D^2 - 4D + 3)y = \sin 3x + x^2$. (8)

(ii) Solve : $y'' + \frac{1}{x} y' = \frac{12 \log x}{x^2}$. (8)

Or

(b) (i) Solve : $\frac{dx}{dt} + y = \sin t + 1$,

$\frac{dy}{dt} + x = \cos t$

given that $x = 1$, $y = 2$ when $t = 0$. (10)

(ii) Solve : $(D^2 - 4D + 4)y = x \sin x$. (6)