Q 9320

B.Sc. DEGREE EXAMINATION, MAY/JUNE 2006.

Second Semester

Computer Technology / Apparel and Fashion Technology

BCS 122 – ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Regulations 2003)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^2 \int_0^y xy \, dx \, dy$.

2. Evaluate $\int_0^{2\pi} \int_0^a r \, dr \, d\theta$.

3. Find div $\vec{A}$ if $\vec{A} = xi + 7yj - 5xyk$.

4. State Gauss divergence theorem.

5. Find the direction cosines of the line joining the points (1, 1, –2) and (2, 3, 0).

6. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 2y - 8z - 9 = 0$.

7. Is there any analytic function with $u = x^2 + y^2$ as the real part? If so, find that analytic function.

8. State the Cauchy-Riemann equations for $P(r, \theta) + iQ(r, \theta)$ to be analytic.

9. Find the residue at $z = 2$ for the function $f(z) = \frac{3z + 1}{z^2 + 3z - 10}$.

10. State Cauchy residue theorem.

PART B — (5 × 16 = 80 marks)

11. (i) Show that the real and imaginary parts of an analytic function are harmonic. (8)

(ii) Find the analytic function whose imaginary part is $e^{-x}(y\cos y - x\sin y)$. (8)

12. (a) (i) Evaluate $\iint x\,dx\,dy$ over the finite area bounded by $y^2 = x$ and $y = x$. (8)

(ii) Find the volume of a sphere of radius a using triple integration. (8)

Or

(b) (i) Evaluate $\int_0^2 \int_0^{4-x} (x+2y)\,dy\,dx$ without changing the order of integration. (8)

(ii) Evaluate $\int_0^2 \int_x^{4-x} (x+2y)\,dy\,dx$ by changing the order of integration. (8)

13. (a) (i) Find the directional derivative of $xz + 4y^2$ at (1, −2, 3) along the direction $6i - 3j + 2k$. (8)

(ii) Using Gauss divergence theorem evaluate $\iint_S \vec{A}\cdot\vec{n}\,dS$ where $\vec{A} = xi + yj + zk$ and $\vec{n}$ is the outward unit normal to the surface S bounded by $x = 0, x = 2, y = 0, y = 2, z = 0$ and $z = 2$. (8)

Or

(b) (i) Prove that $2xyi + (x^2 + 3y^2 z)\,j + y^3\,k$ is irrotational and find its scalar potential. (8)

(ii) Find curl curl $\vec{F}$ at (3, 2, 1) where $\vec{F} = x^3 i + y^2 j + zk$. (8)

14. (a) (i) Find the equation of the plane passing through the points $(2, -1, 2)$, $(1, 1, 1)$ and $(6, 3, -2)$. (8)

(ii) Find the shortest distance between the lines

$$\frac{x-1}{6}=\frac{y-2}{2}=\frac{z-3}{1} \text{ and } \frac{x-2}{4}=\frac{y}{1}=\frac{z-1}{1}.$$ (8)

Or

(b) (i) Find the equation of the plane passing through the intersection of $x+y+2z=7$ and $2x-y+z=6$ and perpendicular to $2x+6y-z=5$. (8)

(ii) Find the centre and radius of the circle.

$$x^2+y^2+z^2-2x-4y-6z-11=0,\ 2x-2y+z+8=0.$$ (8)

15. (a) (i) Find the residues of $\frac{1}{z^4-16}$ at its poles. (8)

(ii) Using Cauchy integral formula evaluate

$$\int_c \frac{z}{(z-3)^2(z-1)}\,dz \text{ where C, is } |z| = 2.$$ (8)

Or

(b) (i) Expand $f(z)=\frac{2z-5}{z^2-5z+4}$ as Laurant series valid in $1 < |z| < 4$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{97+72\cos\theta}$ using contour integration. (8)
